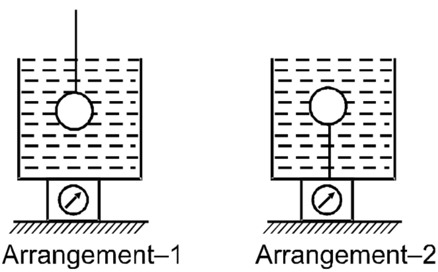
Q-1 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A container open from top, filled with water (density ρ_w) upto the top, is placed on a weighing machine and the reading is W. A wooden ball of volume V and mass m is put in the water by the given two arrangements. In arrangement1, the ball is connected by a rigid rod (of negligible volume) and pushed in the water. In arrange- ment2, the ball is attached with bottom by a massless string. The reading of weighing machine, (density of wood is less than water) (choose incorrect option)



(A) In arrangement-1 is W

(B) in arrangement -1 is $W + \rho V g$

(C) In arrangement-2 is $W + mg - \rho_w V g$

(D) In arragement-2 is less that in arragement -1

Correct Option : B

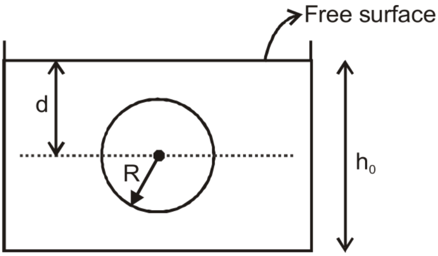
SOLUTION

(1) In arrangement -1, water of weight $\rho V g$ gas come out, but the buyoancy force is also equal to the weight of displaced liquid. So, reading of weighing machine is W. (2) in arrangement-2, weight of the ball mg is added, but water of weight $\rho_w V g$ is removed so reading of weighing machine is W+ mg -$\rho V g$.

Q-2 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A uniform solid sphere of radius R is in equilibrium inside a liquid whose density varies with depth from free surface as

$\rho = \rho_0\left(1+\dfrac{h}{h_0}\right)$ where h is depth from free surface. Density of sphere σ will be :



(A) $\sigma = \rho_0\left(1+\dfrac{d}{2h_0}\right)$

(B) $\sigma = \rho_0\left(1-\dfrac{d}{2h_0}\right)$

(C) $\sigma = \rho_0\left(1+\dfrac{2d}{h_0}\right)$

(D) $\sigma = \rho_0\left(1+\dfrac{d}{h_0}\right)$

Correct Option : D

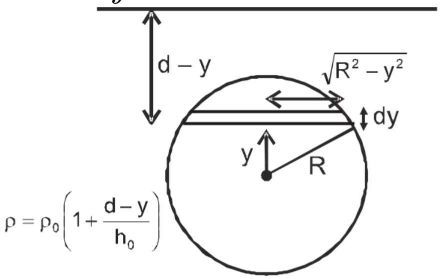
SOLUTION

$$dB = \pi\left(R^2-y^2\right)dy\rho_0\left(1+\frac{d-y}{h_0}\right)g$$

$$dB = \frac{\pi\rho_0 g}{h_0}\left(R^2-y^2\right)\left(h_0+d-y\right)dy$$

$$= \frac{\pi\rho_0 g}{h_0}\left[R^2(h_0+d)dy - R^2ydy - (h_0+d)y^2dy + y^3dy\right]$$

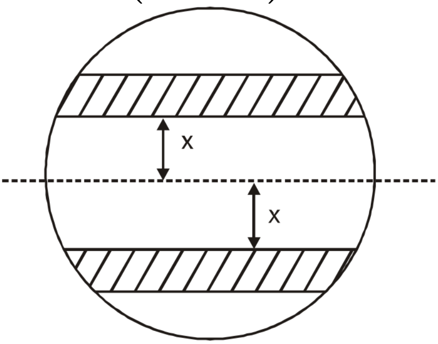
$$B=\int_{y=-R}^{+R} dB=\frac{\pi\rho_0 g}{h_0}\left(R^2(h_0+d)y-\frac{R^2y^2}{2}-(h_0+d)\frac{y^3}{3}+\cdot\right.$$



$$B=\frac{\pi\rho_0 g}{h_0}\left[(h_0+d)R^2(2R)-\frac{(h_0+d)}{3}(2R^3)\right]=\frac{\pi\rho_0 g}{h_0}\left[\frac{4}{3}(h_0+\right.$$

$$=\frac{4}{3}\pi R^3\frac{g(\rho_0)}{h_0}(h_0+d)=\frac{4}{3}\pi R^3 g\sigma\Rightarrow\sigma=\frac{\rho_0}{h_0}(h_0+d)$$

$$\sigma=\rho_0\left(1+\frac{d}{h_0}\right)$$



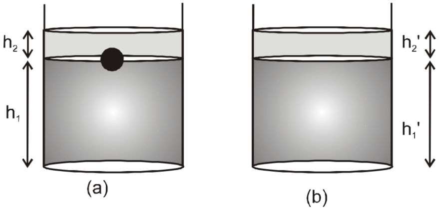
Alternate solution

$$\sigma vg=\int\left[\rho_0\left(1+\frac{d-x}{h_0}\right)dvg+\rho_0\left(1+\frac{d+x}{h_0}\right)dvg\right]$$

$$\sigma v=2\rho_0\left(1+\frac{d}{h_0}\right)\int_0^{v/2}dV=\rho_0 v\left(1+\frac{d}{v_0}\right)$$

$$\Rightarrow\sigma=\rho_0\left(1+\frac{d}{h_0}\right)$$

A piece of Ice floats in a vessel with water above which a layer of lighter oil is poured. When ice melts



1. The level of oil water interface falls , 2. The level of oil water interface rises

3. The thickness of oil layer decreases , 4. The thickness of oil layer remain same

5. The thickness of oil layer increases , 6. The level of oil-air interface falls

7. The level of oil-air interface remains same , 8. The level of oil-air interface rises

Select the correct alternatives :

(A) Only 4 & 7 are correct

(B) 2,3 & are correct

(C) 1,5,7 are correct

(D) only is correct

Correct Option : B

SOLUTION

From fig. (a) h_2A=volume of oil +some volume of ice From fig. (b)

h_2 A =volume of oil

$\Rightarrow (h_2 - h'_2)$ A =Some volume of ice >0

$\Rightarrow h_2 > h'_1$

$\therefore$ Statement 3 correct

Pressure at bottom in fig. (a) is given by

$\Rightarrow P_0 + \rho_{oil}h_2g + \rho_{\text{water}}h_1g$

$\therefore (P_0 + \rho_{oil}h_2g + \rho_{\text{water}}h_1g)A = P_0A + W_{oil} + W_{\text{water}} + W_{\text{Ice}}(i)$

similarly from fig. (b)

$(P_0 + \rho_{oil}h'_2g + \rho_{\text{water}}h'_2g + \rho_{\text{water}}h'_1g)A = P_0A + W_{oil} + W_{\text{wate}}$

$\rho_{oil}(h_2 - h'_2) + \rho_{\text{water}}h'_1 = \rho_{\text{oil}}h_2 + \rho_{\text{water}}h_1$

$$\Rightarrow \rho_{\text{oil}}(h_2 - h'_2) = \rho_{\text{water}}(h'_1 - h_1)$$

$$\Rightarrow h'_1 - h_1 = \frac{\rho_{oil}}{\rho_{\text{water}}}(h_2 - h'_2) > 0$$

$\therefore$ Statement 2 is correct.

Now fall in level $= |h_2 - h'_2|$

and rise in level $= |h'_1 - h_1|$

$$= \frac{\rho_{oil}}{\rho_{\text{water}}}(h_2 - h'_2) < h_2 - h'_2$$

$\Rightarrow$ Fall is more

Statement b is correct

Q-4 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A large open tank is filled with water upto a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to H/n(n>1) and it takes T_2 time to take out the remaining water. If $T_1 = T_2$, then the value of n is

(A) 2

(B) 3

(C) 4

(D) $2\sqrt{2}$

Correct Option : C

SOLUTION

$$-A\frac{dy}{dt} = a\sqrt{2gy}$$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{H} - \sqrt{\frac{H}{n}}\right) = T_1$$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{\frac{H}{n}} - 0\right) = T_2$$

$T_1 = T_2$

n=4.

Q-5 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A capillary tube with inner cross-section in the form of a square of side a is dipped vertically in a liquid of density ρ and surface tension ρ which wet the surface of capillary tube with angle of contact θ . The approximate height to which liquid will be raised in the tube is :

(Neglect the effect of surface tension at the corners of capillary tube)

(A) $\frac{2\sigma \cos\theta}{a\rho g}$

(B) $\frac{4\sigma \cos\theta}{a\rho g}$

(C) $\frac{8\sigma \cos\theta}{a\rho g}$

(D) None of these

Correct Option : B

SOLUTION

Upward force by capilary tube on top surface of liquid is

$f_{up} = 4\sigma a \cos\theta$

If liquid is raised to a height h then we use

$$4\sigma\cos\theta = ha^2\rho g \text{ or } h = \frac{4\sigma\cos\theta}{a\rho g} \text{ Ans.}$$

Q-6 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A sphere of mass m and radius r is projected in a gravity free space with speed v. If coefficient of viscosity is $\frac{1}{6\pi}$ the distance travelled by the body before it stops is

(A) $\frac{mv}{2r}$

(B) $\frac{2mv}{r}$

(C) $\frac{mv}{r}$

(D) None of these

Correct Option : C

SOLUTION

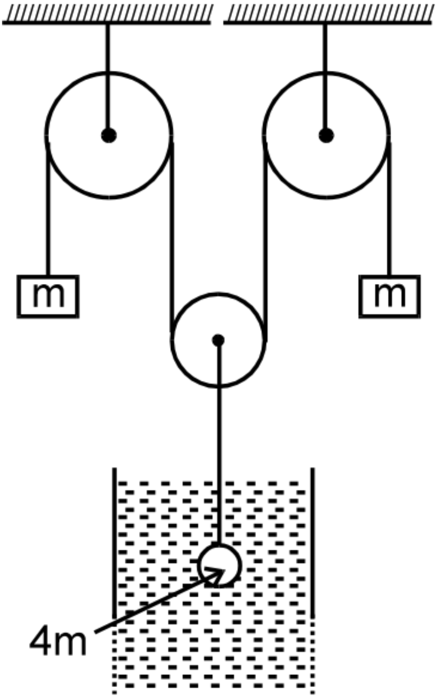
The only force acting on the body is the viscous force

Here $m\frac{vdv}{dx} = -6\pi\eta rv = -rv$

$$\Rightarrow \int_v^0 mdv = \int_0^x -rdx \Rightarrow x = \frac{mv}{r}$$

Q-7 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A spherical ball of mass 4m, density $\sigma$and radius r is attached to a pulley-mass system as shown in figure. The ball is released in a liquid of coefficient of viscosity $\rho\left(<\frac{\sigma}{2}\right)$. If the length of the liquid column is sufficiently long, the terminal velocity attained by the ball is given by (Assume all pulley to be massless and string as massless and inextensible):



(A) $\dfrac{2}{g(r^2(2\sigma-\rho)g)}/(\eta)$

(B) $\dfrac{2}{g(r^2(\sigma-2\rho)g)}/(\eta)$

(C) $\dfrac{2}{g(r^2(\sigma-4\rho)g)}/(\eta)$

(D) $\dfrac{1}{g(r^2(\sigma-2\rho)g)}/(\eta)$

Correct Option : D

SOLUTION

From the free body diagrame of the sphere :

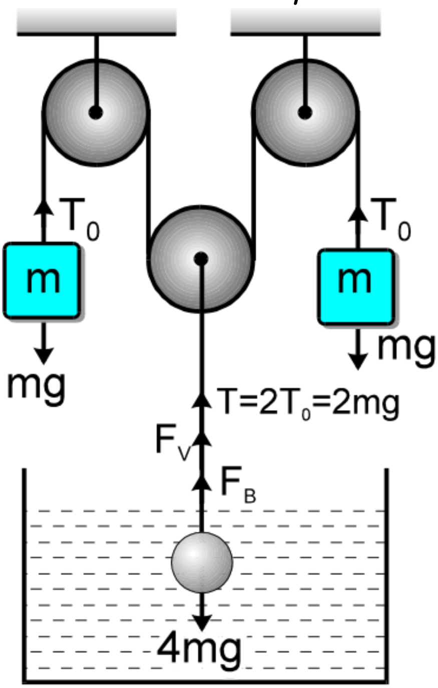
$$F_v = 4mg - 2mg - F_B$$

$$\Rightarrow F_v = 2mg - F_B$$

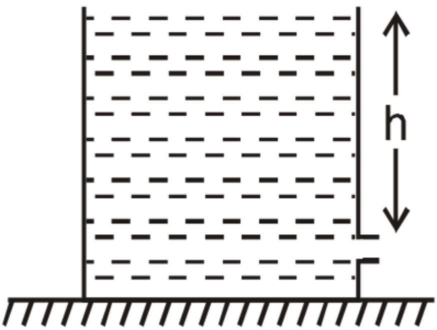
$$\Rightarrow 6\pi\eta rV = \frac{4}{3}\pi r^3\left(\frac{\sigma}{2} - \rho\right)g$$

(since $4\pi = \frac{4}{3}\pi r^3 \times \sigma$)

$$\Rightarrow V = \frac{1}{9}\frac{r^2(\sigma - 2\rho)g}{\eta}$$



In the figure shown, a light container is kept on a horizontal rough surface of coefficient of friction $\mu = \frac{Sh}{V}$. A very small hole of area S is made at depth \'h\'. Water of volume \'V\' is filled in the container. The friction is not sufficient to keep the container at rest. The acceleration of the container initially is



(A) $\frac{V}{Sh}g$

(B) g

(C) zero

(D) $\frac{Sh}{V}g$

Correct Option : D

SOLUTION

Let the density of water be ρ, then the force by escaping liquid on container $= \rho S\left(\sqrt{2gh}\right)^2$

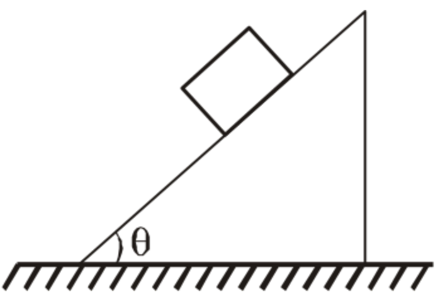
$\therefore$ acceleration of container $a = \dfrac{2\rho Sgh - \mu\rho Vg}{\rho V} = \left(\dfrac{2Sh}{V} - \mu\right)g$

Now $\mu = \dfrac{Sh}{V}$ $\therefore A = \dfrac{Sh}{V}g$

ATTEMPT FREE TEST ON DOUBTNUT

Q-9 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A cubical block of side \' a \' and density \' ρ \' slides over a fixed inclined plane with constant velocity \' v \' . There is a thin film of viscous fluid of thickness \' t \' between the plane and the block . Then the coefficient of viscosity of the thin film will be : (Acceleration due to gravity is g)



(A) $\eta=\dfrac{\rho agt\sin\theta}{v}$

(B) $\dfrac{\rho agt^{2}\sin\theta}{v}$

(C) $\dfrac{v}{\rho agt\sin\theta}$

(D) None of these

Correct Option : A

SOLUTION

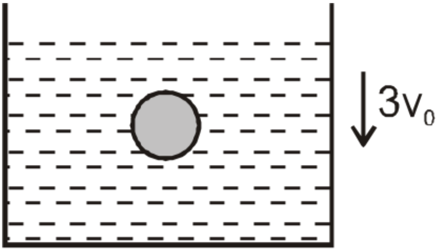
Viscous force $=mg\sin\theta$

$\therefore EtaA\dfrac{v}{t}=mg\sin\theta$ or $\eta a^{2}\dfrac{v}{t}=a^{3}\rho g\sin\theta$

$\eta=\dfrac{t\rho g\sin\theta a}{v}$

Q-10 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A container filled with viscous liquid is moving vertically downwards with constant speed $3v_0$. At the instant shown, a sphere of radius r is moving vertically downwards (in liquid) has speed v_0. The coefficient of viscosity is η. There is no relative motion between the liquid and the container. Then at the shown instant, the magnitude of viscous force acting on sphere is



(A) $6\pi\eta r v_0$

(B) $12\pi\eta r v_0$

(C) $18\pi\eta r v_0$

(D) $24\pi\eta r v_0$

Correct Option : B

SOLUTION

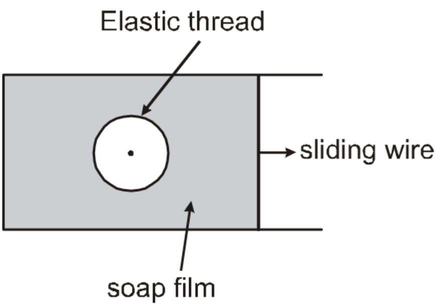
Relative to liquid, the velocity of sphere is $2v_0$ upwards.

$\therefore$ viscous force on sphere $= 6\pi\eta r 2v_0$ downwards

$= 12\pi\eta r v_0$ downwards

Q-11 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

The figure shows a soap film in which a closed elastic thread is lying. The film inside the thread is pricked. Now the sliding wire is moved out so that the surface area increases. The radius of the circle formed by elastic thread will



(A) increases

(B) decreases

(C) remains same

(D) data insufficient

Correct Option : C

SOLUTION

The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

Q-12 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

An isolated and charged spherical soap bubble has a radius r and the pressure inside is atmospheric. If T is the surface tension of soap solution, then charge on drop is:

(A) $2\sqrt{\frac{2rT}{\varepsilon_0}}$

(B) $8\pi r\sqrt{2rT\varepsilon_0}$

(C) $8\pi r\sqrt{rT\varepsilon_0}$

(D) $8\pi r\sqrt{\dfrac{2rT}{\varepsilon_0}}$

Correct Option : B

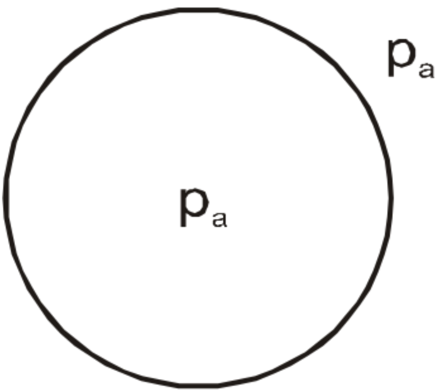
SOLUTION

(B) Inside pressure must be $\dfrac{4T}{r}$ greater than outside outside pressure is provided by charge on bubble.

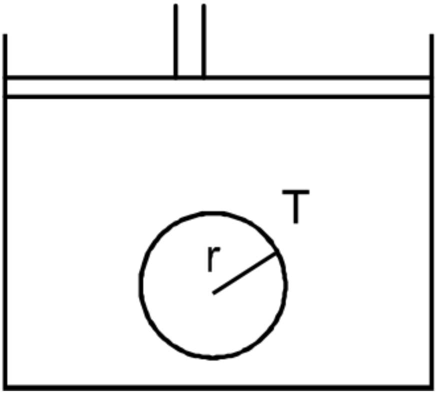
$$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0} \ldots\ldots.\left[\sigma = \frac{Q}{4\pi r^2}\right]$$

$$Q = 8\pi r\sqrt{2rT\varepsilon_0}$$



In a cylinder-piston arrangement, air is under a pressure P_1. A soap bubble of radius r lies inside the cylinder,soap bubble has surface tension T. The radius of soap bubble is to be reduced to half, The new pressure P_2 to which air should be compressed isothermally. (Assume r is very small as compared to height of cylinder)



(A) $P_1 + \frac{4T}{r}$

(B) $4P_1 + \frac{12T}{r}$

(C) $8P_1 + \frac{24T}{r}$

(D) $P_1 + \frac{2T}{r}$

Correct Option : C

SOLUTION

Isothermal process.

$$\left(P_1 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_2 + \frac{4T}{r/2}\right)\left(\frac{4}{3}\pi (r/2)^3\right)$$

$$P_2 = 8P_1 + \frac{24T}{r}$$

Q-14 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

The radius of soap bubble is R and surface tension of soap solution is T, keeping the temperature constant, the extra energy needed to double the radius of the soap bubble by blowing will be

(A) $32\pi R^2 T$

(B) $24\pi R^2 T$

(C) $16\pi R^2 T$

(D) $8\pi R^2 T$

Correct Option : B

SOLUTION

Given :

Initial radius of soap bubble =R

surface tension of soap solution =T

Final radius of soap bubble =2R

The initial energy needed to blow the soap bubble is

$$E_1 = 2 \times 4\pi R^2 \times T = 8\pi R^2 T$$

and final energy needed to blow the soap bubble is

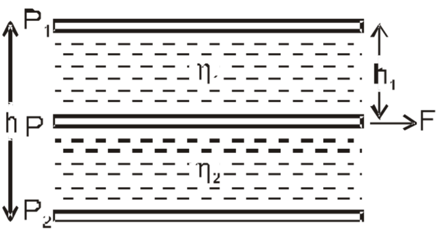
$$E_2 = 2 \times 4\pi (4R)^2 = 32\pi R^2 T$$

Hence extra energy is needed is given by

$$E_2 - E_1 = 32\pi R^2 T - 8\pi R^2 T = 24\pi R^2 T$$

Q-15 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A thin horizontal movable plate P is separated from two fixed horizontal plates P_1 and P_2 by two highly viscous liquids of coefficient of viscosity η_1 and η_2 as shown, where $\eta_2 = 4\eta_1$. Area of contact of movable plate with each fluid is same. If the distance between two fixed plates is h, then the distance h1 of movable plate from upper fixed plate such that the movable plate can be moved with a constant velocity by applying a minimum constant horizontal force F on movable plate is (assume velocity gradient to be uniform in each liquid).



(A) h/4

(B) h/2

(C) 2h/3

(D) h/3

Correct Option : D

SOLUTION

Let v be the velocity of the movable plate and F is equal to viscous force

$$F = \left[\eta_1 \frac{v}{h_1} + \eta_2 \frac{v}{h - h_1}\right] A \Rightarrow \frac{dF}{dh_1} = 0 \therefore h_1 = \frac{h}{3}$$

Q-16 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A container of large uniform cross sectional area A resting on a horizontal surface holds two immiscible non-viscous and incompressible liquids of density d and 3d each of height H/2. The lower density liquid is open to the atmosphere having pressure P_o. A tiny hole of area a $(a < \ < A)$ is punched on the vertical side of the lower container at a height $h\left(0 < h < \frac{H}{2}\right)$ for which range is maximum.

(A) h=H/3

(B) Range R=2H/3

(C) Range R=3H/2

(D) velocity of efflux r $v=\sqrt{\frac{2}{3}gH}$

Correct Option : A

SOLUTION

A,B,D

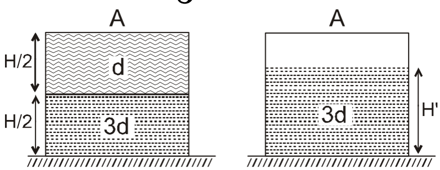
$$\frac{H}{2}\times d+\frac{H}{2}\times 3d=H\times 3d$$
$$\Rightarrow H=\frac{2H}{3}$$
$$V_{\text{efflux}}=\sqrt{2g(H'-h)}$$

V_{efflux} is maximum when h=H\'/2

$$\therefore V_{\max}=\sqrt{\frac{2gH}{3}}$$

Range $R=V_{\text{efflux}}\times\sqrt{\frac{2(H'-h)}{g}}$

$$R_{\max} = \frac{2H}{3}$$



Q-17 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

In a certain gravity free space, the piston of an injection is being pushed so that the water jet comes out with a speed v. The area of the piston is much greater than the orifice of the injection.

(A) The force required to be applied on the piston is proportional to v^2.

(B) The power developed by the force pushing the piston is proportional to v^3.

(C) The time for emptying the injection is proportional to v^{-1}.

(D) The total work done in emptying the injection is proportional to v^2.

Correct Option : A

SOLUTION

$$\frac{F}{A}+\frac{1}{2}\rho v'^2=\frac{1}{2}\rho v^2 \text{ (i)}$$

$$A'v'=Av(ii)$$

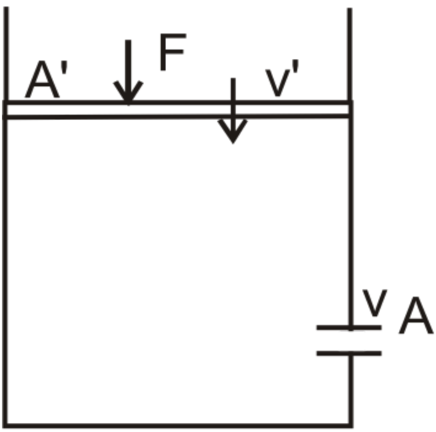
$$\therefore F \propto v^2(A)$$

$$P=F.v'(B)$$

Av=volume flow rate =volume/t

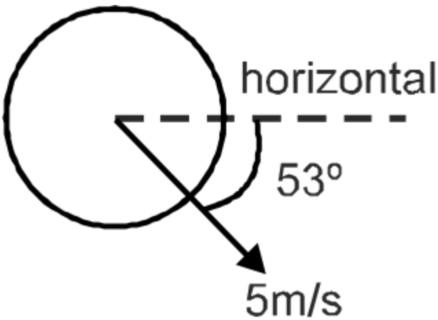
$$\therefore t \propto \frac{1}{v}(C)$$

$$W.D.=\Delta K \Rightarrow (D)$$



Q-18 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

An external force 6N is applied on a sphere of radius R = 10 cm of mass 1 kg and the sphere moves in a liquid with a constant velocity 5 m/s making 530 with the horizontal.The coefficient of viscosity of the liquid is $20/(6\pi)$, in S.I units. (Take $g = 10m/s^2$)



(A) The viscous force on the body is 10 N

(B) The effective weight (weight-upthrust) of the body is 8N

(C) The direction of the external applied force must be horizontal.

(D) If the external force is suddenly removed the acceleration of the body just after the removal of the force will be $6m/s^2$.

Correct Option : A

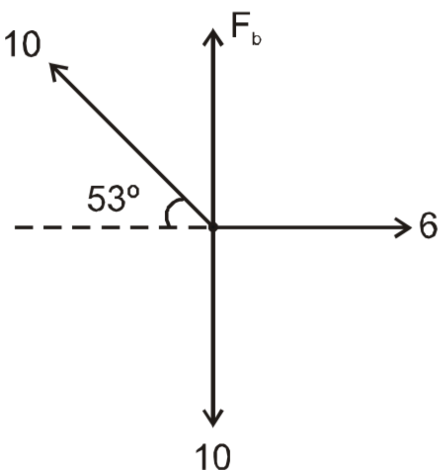
SOLUTION

$F_{\text{drag}} = 6\pi\eta RV$

$= 6\pi\frac{20}{6\pi} \times 0.1 \times 5 = 10N$

$F_b + 8 = 10$

$F_b = 2$



A block of density 2000 kg/m^3 and mass 10 kg is suspended by a spring of stiffness 100 N/m. The other end of the spring is attached to a fixed support. The block is completely submerged in a liquid of density 1000 kg/m^3. If the block is in equilibrium position (g = 10m/s^2).

(A) the elongation of the spring is 1 cm

(B) the magnitude of buoyant force acting on the block is 50 N

(C) the spring potential energy is 12.5 J

(D) magnitude of spring force on the block is greater than the weight of the block

Correct Option : B

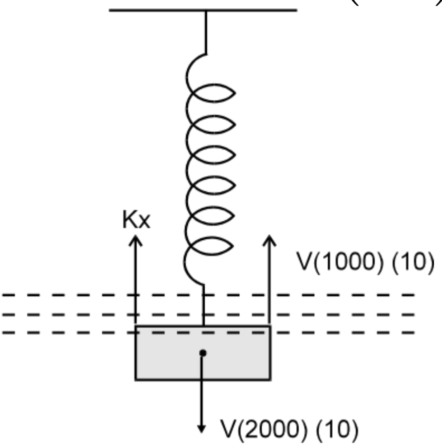
SOLUTION

Kx=V(2000)(10)-V(1000)(10)

$$= \frac{10}{20000}[1000 \times 10]$$

$$Kx = 50N.... (b)$$

$$U_{\text{stored}} = \frac{1}{2} \times (100)\left(\frac{50}{100}\right)^2 = \frac{1}{2} \times \frac{2500}{100} = 12.5J$$



Q-20 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

Lower end of a capillary tube of radius 10^{-3} m is dipped vertically into a liquid. Surface tension of liquid is 0.5 N/m and specific gravity of liquid is 5. Contact angle between liquid and material of capillary tube is 120. Choose the correct options (use g = 10m/s^2)

(A) Maximum possible depression of liquid column in the capillary tube is 1 cm.

(B) Maximum possible depression of mercury column in the capillary tube is 2 cm.

(C) If the length of the capillary tube dipped inside mercury is half of the maximum possible depression of mercury column in the capilary tube, angle made by the mercury surface at the end of the capillary tube with the verticle, is $\cos^{-1}\left(-\frac{1}{4}\right)$.

(D) If the length of the capillary tube dipped inside mercury is one third of the maximum possible depression of mercury column in the capilary tube, angle made by the mercury surface at the end of the capillary tube with the vertical, is $\cos^{-1}\left(-\frac{1}{6}\right)$

Correct Option : A

SOLUTION

$$S = 0.5N/m = r = 10^{-3} m\theta_c = 120\rho = 5 \times 10^3 kg/m^3$$

$$h_{\max} = \frac{2S\cos\theta_c}{r\rho g} = \frac{(2)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{\left(10^{-3}\right)\left(5 \times 10^{-3}\right)(10)} = 10^{-2} m = 1cm$$

If $h = \frac{h_{\max}}{2}$

$$\frac{2S\cos\theta}{r\rho g} = \frac{1}{2}\frac{2S\cos\theta_c}{r\rho g}$$

$$\Rightarrow \cos\theta = -\frac{1}{4}$$

$$\theta = \cos^{-1}\left(-\frac{1}{4}\right)$$

If $h = \frac{h_{\max}}{3}$

$$\frac{2S\cos\theta}{r\rho g} = \frac{1}{3}\frac{2S\cos\theta_c}{r\rho g}$$

$$\Rightarrow \cos\theta = -\frac{1}{6}, \theta = \cos^{-1}\left(-\frac{1}{6}\right)$$

Q-21 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

When a capillary tube is immersed into a liquid, the liquid neither rises nor falls in the capillary ?

(A) The angle of contact must be 90°

(B) The angle of contact may be 90°

(C) The surface tension of liquid must be zero

(D) The surface tension of liquid may be zero

Correct Option : B

SOLUTION

$$h = \frac{2T\cos\theta}{\rho g r}$$

Q-22 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A tank is filled by water ($\rho = 10^3 kg/m^3$). A small hole is made in the side wall of the tank at depth 10 m below water surface. A water jet emerges horizontal from the hole and falls at horizontal distance R from it. The amount of extra pressure (in terms of atmospheric pressure) that must be applied on the water surface, so that range becomes 3R on the on the ground will be (cross section area of hole is negligible and 1 atm = 105 Pa, g = 10 m/s^2)

(A) 8

(B) 11

(C) 7

(D) 5

Correct Option : A

SOLUTION

$\rho gh = 1/2\rho v_1^2 ...(1)$

$\Delta P=\rho g h=1 / 2 \rho v_{2}^{2} \ldots . .(2)$

$v_{2}=3 v_{1} \Rightarrow v_{2}^{2}=9 v_{1}^{2}$

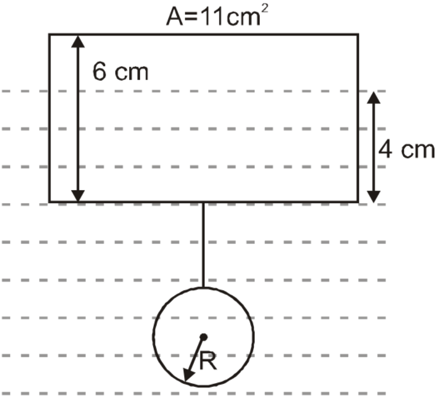
$\Rightarrow \frac{1}{2} \rho v_{2}^{2}=9\left(\frac{1}{2} \rho v_{1}^{2}\right) \Rightarrow \Delta P+\rho g h=9 \rho g h$

$\Delta P=8 \rho g h=8 \times 10^{3} \times 10 \times 10=8 \times 10^{5}$ pascal =8 atm

Q-23 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

Figure shows a uniform metal ball suspended by thread of negligible mass from an upright cylinder that floats partially submerged in water. The cylinder has height 6 cm, face area 11 cm2 on the top and bottom and density 0.5 g/cm^{3}. 4 cm of cylinders height is inside the water surface. Density of the metal ball is 8gm/cm^{3}. R is the radius of the ball. It is found that $R^{3}-\frac{3}{\alpha} cm^{3}$ where α is an integer. Find

α. $\left(\rho_w = 1gm/cm^3\right)$ (system is in equilibrium)



(A) 2

(B) 4

(C) 6

(D) 8

Correct Option : D

SOLUTION

Taking cylinder and the ball as system

$$\frac{4}{3}\pi R^3.\,\rho_2.\,g + Ah.\,\rho_1 g = \frac{4}{3}\pi R^3.\,\rho_w.\,g + Ah_1.\,\rho_w g$$

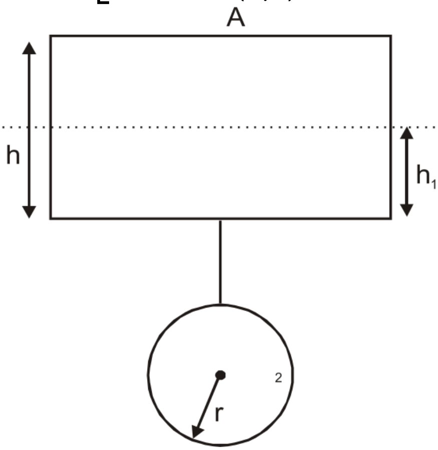
$$\rightarrow R = \left[\frac{3A(h_1\rho_w - h\rho_1)}{4\pi(\rho_2 - \rho_w)}\right]^{1/3}$$

using values

$$A = 11 cm^2, h_1 = 4 cm, \rho_w = 1 gm / cm^3,$$

$$\rho_1 = 0.5 gm / cm^3, \rho_2 = 8 gm / cm^3$$

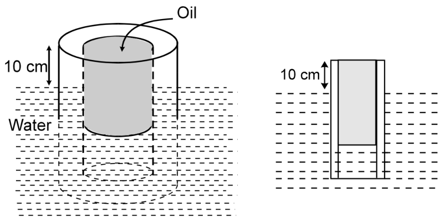
$$R = \left[\frac{3 \times 11(4 \times 1 - 6 \times 0.5)}{4 \times \left(\frac{22}{7}\right) \times (8 - 1)}\right]^{1/3} = \left(\frac{3}{8}\right)^{1/3} cm \Rightarrow R^3 = 3/8$$



Q-24 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A tube with both ends open floats vertically in water. Oil with a density 800 kg/m^3 is poured into the tube. The tube is filled with oil upto the top end while in equilibrium. The portion out of the water is of length 10 cm. The length of oil in the tube is $10\alpha cm$. Find α

(assume effect of surface tension is neglible):



(A) 1

(B) 2

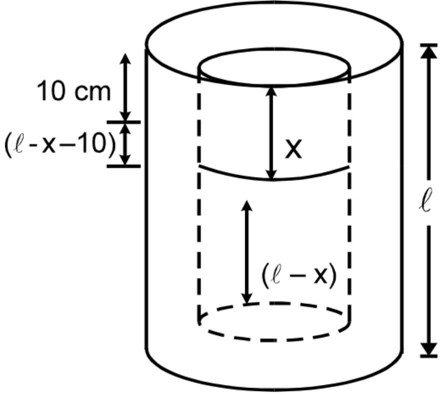
(C) 3

(D) 5

Correct Option : D

SOLUTION

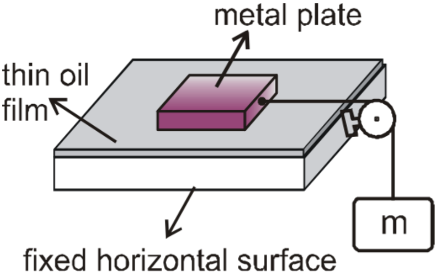
After oil is filled up, pressure at the depth of lower and shouled equate if measured from inside and outside the tube. Suppose depth of oil is x cm then :



$$1000.\, g[(l-10)cm] = 800g.\,(xcm) + 1000g[(l-x)cm] \Rightarrow x = 50$$

Q-25 - JEE ADVANCED-PART TEST-19 (PHYSICS)-PHYSICS

A rectangular metal plate has dimensions of 10 cm 20 cm. A thin film of oil separates the plate from a fixed horizontal surface. The separation between the rectangular plate and the horizontal surface is 0.2 mm. An ideal string is attached to the plate and passes over an ideal pulley to a mass m. When m = 125 gm, the metal plate moves at constant speed of 5 cm/s across the horizontal surface. The coefficient of viscosity of oil in dyne-s/cm^2 is $\frac{\alpha}{2}$. find α (Use g=1000 cm/s^2)



(A) 3

(B) 5

(C) 7

(D) 11

Correct Option : B

SOLUTION

The coefficient of viscosity is the ratio of tangential stress on top surface of film (exerted by block) to that of velocity gradient (vertically downwards) of film. Since mass m moves with constant velocity, the string exerts a force equal to mg on plate towards right.

hence oil shall exert tangential force mg on plate towards left.

$$\therefore \eta = \frac{F / A}{(v - 0) / \Delta x} = \frac{125 \times 1000 / 10 \times 20}{(5 - 0) / 0.2} = 2.5 \text{ dyne-s}/cm^2$$