Q-1 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $2^{\frac{2\pi}{\sin^{-1}x}} - 2(a+2)2^{\frac{\pi}{\sin^{-1}x}} + 8a < 0$for atleast one real x, then

(A) $\frac{1}{8} \leq a < 2$

(B) $a < 2$

(C) $a \in R - \{2\}$

(D) $a \in \left(-\infty, \frac{1}{8}\right) \cup (2, \infty)$

Correct Option : D

SOLUTION

Let $2^{\frac{2\pi}{\sin^{-1}x}} = t \in \left(0, \frac{1}{4}\right] \cup [4, \infty) \Rightarrow (t-2a)(t-4) < 0$

$\therefore 4 < t < 2a \text{ if } 2a > 4 \Rightarrow a > 2$

ALITER: $2a < t < 4$

$$2a < \frac{1}{4}$$

$$a < \frac{1}{8}$$

Q-2 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If p,q are positive integers and \'f\' is a function defined for positive number and attains only positive values, such that $f(xf(y)) = x^p y^q$, then

(A) $p^2 = q$

(B) $p^2 = q^3$

(C) $p = q^2$

(D) $p^2 = q^3$

Correct Option : A

SOLUTION

$$x = \frac{\{f(xf(y))\}^{1/p}}{y^{p/q}}$$

Let $xf(y) = 1 \Rightarrow x = \frac{1}{f(y)} \Rightarrow f(y) = \frac{y^{p/q}}{\{f(1)\}^{1/p}}$

$\Rightarrow f(1) = 1$ (put y=1)

$\Rightarrow f(y) = y^{p/q}$ or $f(x) = x^{q/p}$

Q-3 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

Which of the following is the solution set of the equation

$2\cos^{-1}x = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$?

(A) (0,1)

(B) (-1,1)-{0}

(C) (-1,0)

(D) $(-\infty, -1) \cup [1, \infty]$

Correct Option : A

SOLUTION

$x = \cos\theta$

LHS=2θ where $\theta \in [0, \pi]$

& RHS $= 2\theta$ if $\theta \in [0, \pi/2)$

$= \pi - 2\theta$ otherwise

$\therefore$ RHS=LHS

if $\theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow x \in (0, 1)$

ALITER: Let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta$

$\theta \in [0, \pi]$

Now if $x \in (0, 1) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

LHS=$2\cos^{-1}(\cos\theta) = 2\theta$

RHS=$\cot^{-1}(\cot 2\theta) = 2\theta$

if$x \in (-1, 0) \Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

LHS=$2\cos^{-1}\cos\theta = 2\theta$

RHS=$\cot^{-1}\cot 2\theta = \cot^{-1}\cot(2\theta - \pi) = 2\theta - \pi$

$\therefore$ LHS $\neq$ RHS

Q-4 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

Range of function $f(x) = \left|6\sin^{-1}x - \pi\right| + \left|6\cos^{-1}x - \pi\right|$ is

(A) $[0, 9\pi]$

(B) $[0, 3\pi]$

(C) $[\pi, 3\pi]$

(D) $[\pi, 9\pi]$

Correct Option : D

SOLUTION

$$\begin{cases} -12\sin^{-1}x + 3\pi & x \in \left[-1, \frac{1}{2}\right] \\ \pi & x \in \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \\ 12\sin^{-1} - 3\pi & x \in \left[\frac{\sqrt{3}}{2}, 1\right] \end{cases}$$

Q-5 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

For each positive integer n, let

$f(n+1) = n(-1)^{n+1} - 2f(n)$ and $f(1) = f(2010)$. Then

$\sum_{K=1}^{2009} f(K)$ is equal to

(A) 335

(B) 336

(C) 331

(D) 333

Correct Option : A

SOLUTION

f(2)=1-2f(1)

f(3)=-2-2f(2)

f(4)=3-2f(3)

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f(2010)=2009-2f(2009)

Adding all, 3[f(2)+f(3)+...+f(2009)]+f(2010)+2f(1)=(1+3+...+2009)-(2+4+...+2008)

$\Rightarrow$ 3[f(1)+f(2)+....+f(2009)]=1005

Q-6 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

Number of integral solutions of the equation ,

$$3\tan^{-1}x + \cos^{-1}\left(\frac{1-3x^2}{(1+x^2)^{3/2}}\right) = 0 \text{ is}$$

(A) 1

(B) 2

(C) 0

(D) infinite

Correct Option : B

SOLUTION

$\tan^{-1} x = \theta$

$\therefore 3\theta + \cos^{-1}(\cos 3\theta) = 0$

$\Rightarrow 3\theta \in [-\pi, 0]$

$\Rightarrow x \in \left[-\sqrt{3}, 0\right]$

Q-7 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $e^x + e^{f(x)} = e$,then the range of f(x) is

(A) $(-\infty, 1]$

(B) $(-\infty, 1)$

(C) $(1, \infty)$

(D) $[1, \infty)$

Correct Option : B

SOLUTION

$f(x) = \log_e(e - e^x)$

$\therefore$ for log $(e - e^x)$ to be defined $e - e^x > 0 \Rightarrow y \in (-\infty, 1)$

Q-8 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If f(x)=x+tanx and f(x) is inverse of g(x), then g\'(x) is equal to

(A) $\dfrac{1}{1+(g(x)-x)^2}$

(B) $\dfrac{1}{1+\left(g(x \pm x)\right)^2}$

(C) $\dfrac{1}{2-(g(x)-x)^2}$

(D) $\dfrac{1}{2+(g(x)-x)^2}$

Correct Option : D

SOLUTION

$$f(g(x)) = x \Rightarrow f'(g(x)).\, g'(x) = 1 \Rightarrow g'(x) = \frac{1}{f'(g(x))} = \frac{}{1+}$$

Q-9 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If x and y are of some sign, then the value of

$$\frac{x^3}{2}\cos ec^2\left(\frac{1}{2}\frac{\tan^{-1}x}{y}\right) + \frac{y^3}{2}\sec^2\left(\frac{1}{2}\frac{\tan^{-1}y}{x}\right)$$ is equal to

(A) $(x-y)\left(x^2+y^2\right)$

(B) $(x+y)\left(x^2-y^2\right)$

(C) $(x+y)\left(x^2+y^2\right)$

(D) $(x-y)\left(x^2-y^2\right)$

Correct Option : C

SOLUTION

$$\frac{x^3}{2\sin^2\left(\frac{1}{2}\frac{\tan^{-1}x}{y}\right)}+\frac{y^3}{2\cos^2\left(\frac{1}{2}\frac{\tan^{-1}y}{x}\right)}=\frac{x^3}{1-\cos\left(\frac{\tan^{-1}x}{y}\right)}+\frac{}{1+}$$

$$=\frac{x^3}{1-\frac{|y|}{\sqrt{x^2+y^2}}}+\frac{y^3}{1+\frac{|x|}{\sqrt{x^2+y^2}}}=(x+y)\left(x^2+y^2\right)$$

Q-10 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

In a ΔABC, if

$$\angle A=\angle B=\frac{1}{2}\left(\sin^{-1}\left(\frac{\sqrt{6}+1}{2\sqrt{3}}\right)+\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)\text{ and }c=2.3^3$$

, then

(A) area of $\Delta ABC = 9$ square units

(B) area of $\Delta ABC = 6$ square units

(C) r=2R

(D) R=2r

Correct Option : A

SOLUTION

$\angle A = \angle B = \pi / 3$

Q-11 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If f(x) is identity function, g(x) is absolute value function and h(x) is reciprocal function then

(A) fogoh(x)=hogof(x)

(B) hog(x)=hogof(x)

(C) gofofofohogof(x)=gohog(x)

(D) hohohoh(x)=f(x)

Correct Option : A

SOLUTION

f(x)=x, g(x)=|x|, h(x)=1/x

Q-12 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

The function $y=\frac{x}{1+|x|}:R\to R$ is

(A) one-one

(B) onto

(C) odd

(D) into

Correct Option : A

SOLUTION

$$\frac{dy}{dx} = \frac{1}{(a+|x|)^2} > 0 \Rightarrow \text{ one-one}$$

$$R_f = (-1, 1) \Rightarrow \text{ into}$$

Q-13 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If α, β, γ are roots of equation

$$\tan^{-1}\left(\left|x^2+2x\right| + |x+3| - \left|\left|x^2+2x\right| - |x+3|\right|\right) + \cot^{-1}\Bigg($$

in ascending order $(\alpha < \beta < \gamma)$ then

(A) $\sin^{-1}\gamma$ is defined

(B) $\sec^{-1}\alpha$ is defined

(C) $\gamma - \beta = \sqrt{2}$

(D) $|\beta| > |\gamma|$

Correct Option : A

SOLUTION

$$\tan^{-1}\left(\left|x^2+2x\right|+|x+3|-\left|\left|x^2+2x\right|-|x-3|\ \right|\right)=\pi-\cot^{-}$$

$$=\pi-\left(\pi-\frac{\cot^{-1}1}{2}\right)=\frac{\cot^{-1}1}{2}=\tan^{-1}2$$

(i)

$$\left|x^2+2x\right|\geq|x+3|\Rightarrow 2|x+3|=2\Rightarrow x=\ -2,\ -4\Rightarrow x=\ -$$

(ii)

$$\left|x^2+2x\right|\leq|x+3|\Rightarrow 2\left|x^2+2x\right|=2\Rightarrow x=\ -1,\ -1\pm\sqrt{2}\Rightarrow$$
$$\Rightarrow\alpha=\ -4,\beta=\ -1,\gamma=\ -1+\sqrt{2}$$

Q-14 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If f(x) and g(x) are two polynomial such that the polynomial

$h(x)=xf\left(x^3\right)+x^2g\left(x^6\right)$ is divisible by x^2+x+1, then

(A) f(1)=g(1)

(B) f(1)=-g(1)

(C) h(1)=0

(D) all of these

Correct Option : A

SOLUTION

$h(\omega)=0$ and $h(\omega^2)=0 \Rightarrow \omega f(1)+\omega^2 g(1)=0$ and $\omega^2 f(1)+\omega$

Q-15 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If the solution of equation

$\sin(\tan^{-1}x)=\sqrt{4-[\sin(\cos^{-1}x)+\cos(\sin^{-1}x)]^2}$ is a then

(A) $\sin^{-1}a+\cos^{-1}a=\frac{\pi}{2}$

(B) $2\sin^{-1}a+\cos^{-1}a=\frac{\pi}{2}$

(C) $\sin^{-1}a+3\cos^{-1}a=\frac{3\pi}{2}$

(D) $\tan^{-1}a+\cos^{-1}a=\frac{\pi}{2}$

Correct Option : A

SOLUTION

$$\frac{x}{\sqrt{1+x^2}} = 2|x| \Rightarrow x = 0 \Rightarrow a = 0$$

Q-16 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $f(x) = \frac{2\{x\} - 1}{2\{x\} + 1}$ then (where {x} represent fractional part of x)

(A) $D_f \in R$

(B) $R_f \in \left[0, \frac{1}{3}\right)$

(C) period of f(x) is 1

(D) f(x) is even function

Correct Option : A

SOLUTION

$f(x) = 1 - \frac{2}{2^{\{x\}} + 1}$ and $2 \leq 2^{\{x\}} + 1 < 3$

Q-17 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is a bijective function from set A to set B then which of the following may be true

(A) $A = (-\infty, -1), B = \left(0, \frac{\pi}{2}\right)$

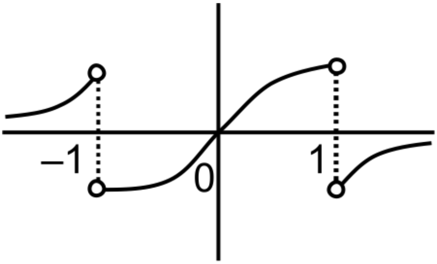
(B) $A = (-1, 1), B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $A = [1, \infty), B = \left(-\frac{\pi}{2}, 0\right]$

(D) all of these

Correct Option : A

SOLUTION



Q-18 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If the functions f(x) and g(x) are defined from R^+ to R such that f(x)= $\begin{cases} 1-\sqrt{x} & xisrational \\ x^2 & xisirrational \end{cases}$ and $g(x) = \begin{cases} x & xisrational \\ 1-x & xisirrational \end{cases}$, then the composite function fog(x) is

(A) one-one

(B) many-one

(C) into

(D) onto

Correct Option : B

SOLUTION

$$\begin{cases} 1-\sqrt{x} & x \in Q \\ (1-x)^2 & x \notin Q \end{cases}$$

$\Rightarrow fog(\sqrt{2}-1) = fog(3-\sqrt{2})$ $\therefore$ many-one alos into

Q-19 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

Let f(x) $= \cot^{-1}(x^2 + 4x + \alpha^2 - 3\alpha)$ be a function defined on $R \to \left(0, \frac{\pi}{2}\right]$, is an onto funtion then

(A) $\alpha \in [-1, 4]$

(B) $\alpha \in \{-1, 4\}$

(C) f(x) is one-one

(D) f(x) is many-one

Correct Option : B

SOLUTION

$f(x) = \cot^{-1}\left((x+2)^2 + \alpha^2 - 3\alpha - 4\right)$. For f(x) to be onto,
$\alpha^2 - 3\alpha - 4 = 0$

Q-20 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

The number of solution of equation $2\cos^{-1}x = a + a^2\left(\cos^{-1}x\right)^{-1}$ are

(A) at least 1 if $a \in [-2\pi, \pi] - \{0\}$

(B) 1 if $a \in (0, \pi]$

(C) 1 if $a \in [-2\pi, 0)$

(D) 2 if $a > 0$

Correct Option : A

SOLUTION

Let $\cos^{-1} x = t \Rightarrow 2t = a + \frac{a^2}{t} \Rightarrow 2t^2 - at - a^2 = 0$

$\Rightarrow t = a, -\frac{a}{2}$ where $t \neq 0$

Q-21 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

The function f: $\left[-\frac{1}{2}, \frac{1}{2}\right] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ defined by f(x)= $\sin^{-1}(3x - 4x^3)$ is

(A) a surjective function

(B) an injective function

(C) a surjective but not injective

(D) neither injective nor surjective

Correct Option : A

SOLUTION

Let

$\sin^{-1}x = \theta, \theta \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \therefore f(x) = \sin^{-1}(\sin 3\theta) = 3\theta = 3\sin$

Q-22 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $f(x) = \left[\frac{1}{\ln(x^2+e)}\right] + \frac{1}{1+x^2}$ where [.] is greater integer function then

(A) $f(x) \in \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \cup \{2\}$ for $x \in R - \{1\}$

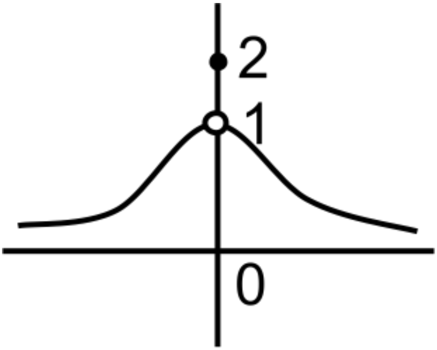
(B) $R_f = (0, 1) \cup \{2\}$

(C) f is many-one

(D) f(x) is bounded

Correct Option : B

SOLUTION



at x=0, f(0)=2 for $x \neq 0$, $f(x) = 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}$

Q-23 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

If $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$, then $\underbrace{h(h(h...(h(x))))}_{hrepeatedn \times}$ is

(A) identity function

(B) one-one

(C) odd

(D) periodic

Correct Option : A

SOLUTION

$f(x) = \begin{cases} 3x & : \geq 0 \\ x & : x < 0 \end{cases}$ and $g(x) = \begin{cases} \frac{x}{3} & : x \geq 0 \\ x & : x < 0 \end{cases}$ $\therefore h(x) = x$

Q-24 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

The function f: $R \to (-1, 1)$ is defined by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) f(x) is a bijective function

(B) f(x) is non-bijective function

(C) $f^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

(D) f(x) is many one onto function

Correct Option : A

SOLUTION

f(x) is strictly incr. function

$$\therefore x \in (-\infty, \infty) \Rightarrow f(x) \in \left(\underset{x \to \infty}{\leq} tf(x), \underset{x \to \infty}{\leq} tf(x) \right) \in (-1, 1)$$

Q-25 - JEE ADVANCED-PART TEST-3 (MATHS)-MATHS

Area bounded by curve $y = \max . \{x + 3y + 1, 2x - y\}$ and line $x = -3$ is equal to

(A) the area bounded by parabola $y^2 = 4x$ and line x=1

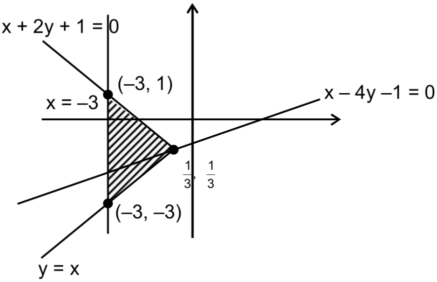
(B) the twice of the area bounded by parabola $y^2 = 4x$ and line x=1

(C) the area between curves $y^2 = 4x$ and $x^2 = 4y$

(D) the half of area bounded by parabola $y^2 = 4x$ and line x=4

Correct Option : B

SOLUTION



Required area $=\frac{1}{2}.\,4.\,\frac{8}{3}=\frac{16}{3}$