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Q-1 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Let f be a continous real function such that f(11)=10 and for all x,f(x)

f(f(x))=1 then f(9) =

(A) 9

(B) $\frac{1}{9}$

(C) $\frac{10}{9}$

(D) $\frac{9}{10}$

Correct Option : B

SOLUTION

$f(f(x)) = \frac{1}{f(x)} \Rightarrow f(y) = \frac{1}{y}$ where y is in the range of f f(11)=10 $\Rightarrow$ 10 is in range of f $\Rightarrow f(10) = \frac{1}{10}$

So 10 and 1/10 are in range of f.

So by IVT, 9 is in range of f $\Rightarrow f(9) = \frac{1}{9}$

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Q-2 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

The value of $\lim_{x \to \infty} \frac{e^{2x} - \cos x - \ln(1 + 2x)}{x \tan x - \sin x}$ is

(A) 0

(B) 1

(C) 2

(D) 3

Correct Option : A

SOLUTION

$$\lim_{x \to \infty} \frac{e^{2x} - \cos x - \ln(1 + 2x)}{\tan x(x - \cos x)} = \lim_{x \to \infty} \frac{e^{2x} - \cos x - \ln(1 + 2x)}{x(x - \cos x)}$$

$$= \lim_{x \to \infty} \frac{\left(1 + \frac{2x}{1!} + \frac{4x^2}{2!} + ...o\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .... . \infty\right) - \left(\right.}{x\left(x - 1 + \frac{x^2}{2!} - \frac{x^4}{4!} .... . \infty\right)}$$

$$= \lim_{x \to \infty} \frac{\frac{9x^2}{2} - \frac{4x^3}{3} + ... \infty}{-x + x^2 + \frac{x^3}{2} + ...\infty} = 0$$

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Q-3 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

The value of $\lim_{x \to 0} x\left[\frac{1}{x}\right]$ where [.] denotes GIF , is

(A) 0

(B) 1

(C) 2

(D) infinite

Correct Option : B

SOLUTION

$$\lim_{x \to \infty} x\left(\frac{1}{x} - \left\{\frac{1}{x}\right\}\right) = \lim_{x \to \infty} \left[1 - x\left\{\frac{1}{x}\right\}\right] = 1 - 0 = 1$$

Aliter : $\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \leq \frac{1}{x} \Rightarrow 1 - x < x\left[\frac{1}{x}\right] \leq 1$

$\lim_{x \to \infty} (1 - x) = 1 = \lim_{x \to \infty} (1)$

so by sandwitch theorem $\lim_{x \to \infty} x\left[\frac{1}{x}\right] = 1$

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Q-4 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

$\lim_{x \to \infty} \left[\sum_{r=1}^{n} \frac{1}{2^r}\right]$ where [.] denotes GIF is equal to

(A) 1

(B) 0

(C) non-existent

(D) 2

Correct Option : B

SOLUTION

$$\lim_{n \to \infty} \left[\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n}\right] = \lim_{n \to \infty} \left[\frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}}\right]$$

$$= \lim_{n \to \infty} \left[1 - \frac{1}{2^n}\right] = 0 as \frac{1}{2^n} \to 0^+$$

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Q-5 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

The function f(x) $= f(x) = \begin{cases} [x^2] + sgn(x-1) & ,x \le 2 \\ \frac{\sin\{2x-3\}}{x-2} & ,x > 2 \end{cases}$ where [.] and {.} denotes GIF and FPF respectively is

(A) continous at x=2

(B) differentiable at x =2

(C) continous and differentiable at x=2

(D) discontinuous at x=2

Correct Option : D

SOLUTION

$$\lim_{x \to 2^-} f(x) = 3 + 1 = 4$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{\sin(2x-4)}{x-2} = 2$$

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Q-6 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Let f: $R^+ \to R^+$ be a monotonic continous function so that there exists at least one $c \in [a, b], a, b > 0$ such that f(c)= $\frac{\lambda f(a) + f(b)}{\lambda + 1}, \lambda > 0$ then possible value(s) of 'c' is given by

(A) $c = f^{-1}\left(\frac{2f(a)f(b)}{f(a) + f(b)}\right)$

(B) $c = f^{-1}\left(\frac{f^2(a)f^2(b)}{f(a) - f(b)}\right)$

(C) $c = f^{-1}\left(\frac{2f(a)f(b)}{f(a) - f(b)}\right)$

(D) $c = f^{-1}\left(\frac{f^3(a)f^3(b)}{f(a) - f(b)}\right)$

Correct Option : A

SOLUTION

Let $f(b) \geq f(a)$

so $\dfrac{2f(a)f(b)}{f(a)+f(b)} \geq f(a) \Rightarrow f(b) \geq f(a)$ true

& $\dfrac{2f(a)f(b)}{f(a)+f(b)} \leq f(b) \Rightarrow f(a) \leq f(b)$ true

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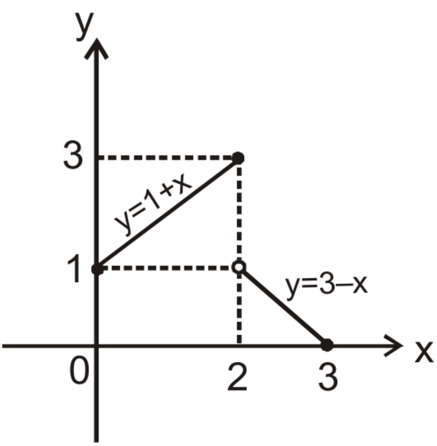
Q-7 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

if if $f(x) = \begin{cases} (1+x) & ,0 \leq x \leq 2 \\ 3-x & ,2 < x \leq 3 \end{cases}$ then the number of values of x at which the function fof is not differentiable is



(A) 2

(B) 1

(C) 3

(D) 0

Correct Option : A

SOLUTION

$$f(f(x))=\begin{cases}1+f(x) & ,0\le f(x)\le 2\\ 3-f(x) & ,2<f(x)\le 3\end{cases}$$

$$=\begin{cases}1+f(x) & ,0\le f(x)<1\\ 1+f(x) & ,1\le f(x)\le 2\\ 3-f(x) & ,2<f(x)\le 3\end{cases}$$

$$=\begin{cases}1+(3-x) & ,2<x\le 3\\ 2+x & ,0\le x\le 1\\ 3-(1+x) & ,1\le x\le 2\end{cases}$$

$$=\begin{cases}x+2 & ,0\le x\le 1\\ 2-x & ,1<x\le 2\\ 4-x & ,2<x\le 3\end{cases}\Leftrightarrow \text{f(x) is non differentiable at x=1,2}$$

f(x),x=1,2

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if $a^2b^2 + bx + c = 0$ has roots α and $\beta(\alpha < \beta)$ and $-p^2x^2 + qx + r = 0$ has roots $\alpha + 5$ and $\beta - 5$ then $(a^2 + p^2)x^2 + (b - q)x + (c - r) = 0$ has a root in interval (given $\alpha + 5 < \beta - 5$)

(A)

(B)

(C)

(D)

Correct Option : A

SOLUTION

$f(x) = (a^2x^2 + bx + c) - (-p^2x^2 + qx + r)$

$f(\alpha) = 0 - (-ve) = +ve$

$f(\alpha + 5) = (-ve) - (0) = -ve \Rightarrow$ root in $(\alpha, \alpha + 5)$

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Q-9 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Let f(x) be a function which is differentiable everywhere any number of times and $f(2x^2-1)=2x^3 f(x)\,\forall x \in R$ then $f^{2010}(0)$ is equal to ($f^n(x)$ is n^{th} derivative of f(x))

(A) -1

(B) 1

(C) 0

(D) Data provided is insufficient

Correct Option : C

SOLUTION

$f(2x^2-1)=-2x^3 f(-x) \Rightarrow 2x^3 f(x)=-2x^3 f(-x)$

$\Rightarrow f(-x)=-f(x) \Rightarrow f(x)$ is odd $\Rightarrow f^{2010}(x)$ is odd

$\Rightarrow f^{2010}(0)=0$

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Q-10 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

If $y = x + e^x$ then $\left(\frac{d^2x}{dy^2}\right)_{x=\ln 2}$ is

(A) $-\frac{1}{9}$

(B) $-\frac{2}{27}$

(C) $\frac{2}{27}$

(D) $\frac{1}{9}$

Correct Option : B

SOLUTION

$$\frac{dy}{dx} = 1 + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x} \Rightarrow \frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^3}$$

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If f(x) $\begin{cases} \text{Minimum}\{x, x^2\}; x \geq 0 \\ \text{Minimum}\{2x, x^2 - 1\}; x < 0 \end{cases}$

then number of points in the interval [-2,2] where f(x) is non-differentiable is:

(A) 1

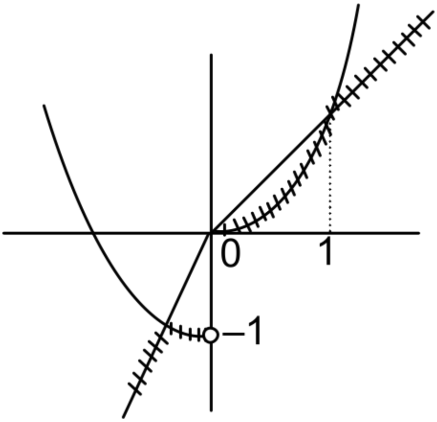
(B) 2

(C) 3

(D) 4

Correct Option : C

SOLUTION



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Q-12 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Let $f''(x)$ be continues at x=0, $f(0) \neq 0$, $f'(0) \neq 0$ and $\lim_{x \to 0} \dfrac{2f(-x) - 3af(2x) + bf(8x)}{\sin^2 x}$ exists, then the value of a+b is :

(A) 0

(B) 2

(C) 1

(D) 3

Correct Option : B

SOLUTION

Let $L = \lim_{x \to 0} \frac{2f(-x) - 3af(2x) + bf(8x)}{\sin^2 x}$

for L to exists

we have 2f(0)-3af(0)+bf(0)=0

$\Rightarrow$ 3a-b=2 (i)

Now, $L = \lim_{x \to 0} \frac{2f'(-x) - 6af'(2x) + 8bf'(8x)}{2x}$

for L to exists, we have

-2f'(0)-6af'(0)+8bf'(0)=0 $\Rightarrow$ 4b-3a=1 ...(ii)

from (i) and (ii) a=b=1

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Q-13 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

if $f(x) = \begin{cases} \frac{(e^x - 1)\,|x \sin x|}{x^n} & ,x > 0 \\ 0 & ,x = 0 \end{cases}$ is derivable at x=0 then interval of possible values of 'n' is

(A) $[1, \infty)$

(B) $(-\infty, 1]$

(C) $(-\infty, 2]$

(D) [1,2]

Correct Option : C

SOLUTION

$$f'(0) = \lim_{x \to 0} \frac{\frac{(e^x - 1)(x \sin x)}{x^n} - 0}{x} = \lim_{x \to 0} \left(\frac{\sin x}{x^{n-1}}\right) = \lim_{x \to 0} \left(\frac{}{x}\right.$$

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Q-14 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Find 'a' if equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root

(A) 0

(B) 1

(C) 2

(D) -2

Correct Option : D

SOLUTION

$$\alpha^4 + a\alpha^2 + 1 = 0 \Rightarrow \alpha(\alpha^3 + a\alpha) + 1 = 0 \Rightarrow -\alpha + 1 = 0$$
$$\Rightarrow \alpha = 1 \Rightarrow 1 + a + 1 = 0$$

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Q-15 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS if f(x)=$ax^2 + bx + c$ such that f(p)+f(q)=0 where $a \neq 0; a, b, c, p, q \in R$ then number of real roots of equation f(x)=0 in interval [p,q] is

(A) Exactly one

(B) at least one

(C) at most one

(D) Data provided is insufficient

Correct Option : B

SOLUTION

f(p)=-f(q) $\Rightarrow$ either $f(p)f(q) < 0$ or f(p)=0=f(q)

$\Rightarrow$ exactly one root in (p,d) or roots are p and q

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If $a = \dfrac{25 - x^2}{16}$ and $\dfrac{\log_a((24 - 2x - x^2))}{14} > 1$, then $x \in$

(A) (0,1)

(B) $(-3, 1) \cup (3, 4)$

(C) $(-\infty, -17) \cup (1, \infty)$

(D) (4,5)

Correct Option : B

SOLUTION

$24 - 2x - x^2 > 0, a > 0, a \neq 1$

$\Rightarrow x^2 + 2x - 24 < 0, x^2 - 25 < 0, x \neq \pm 3$

$\Rightarrow x \in (-5, -3) \cup (-3, 3) \cup (3, 4)$

case (i) : $x \in (-5, -3) \cup (3, 4) \Rightarrow \dfrac{25 - x^2}{16} < 1 \Rightarrow 0 < a < 1$

so given equation becomes

$\dfrac{24 - 2x - x^2}{14} < a \Rightarrow \dfrac{24 - 2x - x^2}{14} < \dfrac{25 - x^2}{16}$

$\Rightarrow x^2 + 16x - 17 > 0 \Rightarrow x \in (-\infty, -17) \cup (1, \infty) \Rightarrow x \in (3,$

case(ii) : $x \in (-3, 3) \Rightarrow a \in \left(1, \dfrac{25}{16}\right]$

so given equation becomes

$$\frac{24-2x-x^2}{14} > a \Rightarrow x \in (-17, 1) \Rightarrow x \in (-3, 1)$$

from case (i) & (ii) we get $x \in (-3, 1) \cup (3, 4)$

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Q-17 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

If $f(x) = \frac{x+3}{5x^2+x-1}$ & $g(x) = \frac{2x+3x^2}{20-2x-x^2}$ such that f(x) and g(x) are differentiable functions in their domains, then which of the following is/are true

(A) 2f'(2)+g'(1)=0

(B) 2f'(2)-g'(1)=1

(C) f'(1)+2g'(2)=0

(D) f'(1)-2g'(2)=1

Correct Option : A

SOLUTION

Since $f\left(\frac{2}{x}\right) = g(x)$

$\therefore f'\left(\frac{2}{x}\right)\left(-\frac{2}{x^2}\right) = g'(x)(1)$

Now on putting x=1 and x=2 in equ.(1) we get

$2f'(2) + g'(1) = 0$ & $f'(1) + 2g'(2) = 0$

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Q-18 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

If f(x) is a polynomial of degree five with leading coefficient one such that f(1)=2, f(2)=8,f(3)=18,f(4)=32,f(5)=50 then

(A) f(6)=192

(B) $\lim_{x \to \infty} \frac{f(x)}{x^5} = 1$

(C) $f(0) < 0$

(D) $f(0) > 0$

Correct Option : A

SOLUTION

$$f(x) - 2x^2 = (x-1)(x-2)(x-3)(x-4)(x-5)$$

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Q-19 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

$$\text{if } f(x) = \begin{cases} x^3 - 6x^2; \text{if x is rational} \\ -11x + 6; \text{if x is irrational} \end{cases}$$

(A) f(x) is continous for all real x

(B) f(x) is continuous at x=1,2,3

(C) f(x) is discontinous at infinite points

(D) f(x) discontinuous for all real x

Correct Option : B

SOLUTION

f(x) is continuous when

$x^3 - 6x^2 = -11x + 6 \Rightarrow x^3 - 6x^2 + 11x - 6 = 0 \Rightarrow x = 1, 2, 3$

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Q-20 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

If x=a satisfies equation

$\tan^{-1}(x+2) + \cot^{-1}\sqrt{4x+20} = \lim_{x \to \frac{\pi}{2}} \frac{\sin(x\cos x)}{\cos(x\sin x)}$ and

$\lim_{x \to 0} \frac{\sqrt[a]{1+x} - \frac{(1+x)^{\frac{1}{x}}}{e} - bx}{5x + kx^2 + x^3} = 0$ then

(A) a=2

(B) ab=3

(C) a=1,b=3

(D) $k \in R$

Correct Option : B

SOLUTION

$$\tan^{-1}(x+2)+\cot^{-1}\sqrt{4x+20}=\frac{\pi}{2}\Rightarrow x+2=\sqrt{4x+20}\Rightarrow x$$

$$\Rightarrow \lim_{x\to 0}\frac{\sqrt[4]{1+x}-\frac{(1+x)^{\frac{1}{x}}}{e}-bx}{5x+kx^2+x^3}=0\Rightarrow \lim_{x\to 0}\frac{\left(\frac{3}{4}-b\right)x-\frac{53}{96}x}{5x+kx^2+}$$

$\underline{\in}\ R$

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Q-21 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

if $f(x)=\begin{cases}\frac{[\tan x]}{[x]} & ,[x]\neq 0\\ 0 & ,[x]=0\end{cases}$ where [.] =GIF, then

(A) $\lim_{x\to 0^-} f(x)=1$

(B) $\lim_{x\to 0^+} f(x)=1$

(C) $\lim_{x\to 0^+} f(x)=0$

(D) f(0)=0

Correct Option : A

SOLUTION

$$f(0)=0=\lim_{x\to 0^+} f(x) \Rightarrow \lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{[\tan x]}{[x]}=\frac{-1}{-1}=$$

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Q-22 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

if $f(x)=\begin{cases}\frac{1}{x^2}-\frac{1}{x^2} & ,x<0 \\ \sin^{-1}(x+b) & ,x\geq 0\end{cases}$ then at x=0, f(x) is

(A) continuous if b=0

(B) discontinuous for any real b

(C) differentiable for b $=\pm 1$

(D) non-differentiable for any real b

Correct Option : A

SOLUTION

Continuous $\Rightarrow 0=\sin^{-1} b \Rightarrow b=0$

Differentiable $\Rightarrow f'(0^-) = f'(0^+) \Rightarrow 0 = 1 \Rightarrow$ not possible

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Q-23 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

Consider a continuous function $f: [0, \infty) \rightarrow [0, \infty)$.if

$f(ab) = f(a)f(b)$ for all a,b in the domain of 'f' and $\lim_{x \rightarrow \infty} f(x)$ is a non-zero finite number then

(A) f(2)=2

(B) $\sum_{r=1}^{10} f(r) = 55$

(C) $\sum_{r=0}^{10} f(r) = 11$

(D) f'(2)=0

Correct Option : C

SOLUTION

Let $\lim_{x \to \infty} f(x) = \lambda(\neq 0)$

For any

$a > 0, \lambda = \lim_{x \to \infty} f(x) = \lim_{x \to \infty} f(ax) = \lim_{x \to \infty} f(a)f(x) = f(a)$

$\Rightarrow f(1) = 1, f(2) = 1$

....

Also $f(0) = \lim_{x \to 0^+} f(x) = 1$

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Q-24 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

If $f(x) = ||\sin(|x| - 1)| - 2|$ then

(A) f(x) is continuous at x=2

(B) f(x) is differentiable at x=2

(C) f'(2)=cos 1

(D) f(x) is non-differentiable at x=0

Correct Option : A

SOLUTION

Around

$x = 2, f(x) = |\sin(x-1) - 2| = 2 - \sin(x-1) \Rightarrow f'(x) = -$ c

Now

$$f(x) = \begin{cases} 2 - \sin(x+1) & ,x \to 0^- \\ 2 + \sin(x-1) & ,x \to 0^+ \end{cases} \Leftrightarrow f'(0^-) = -\cos 1, f'(0^+$$

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Q-25 - JEE ADVANCED-PART TEST-6 (MATHS)-MATHS

A quadratic equation f(x) $= ax^2 + bx + c = 0$ has positive distinct roots reciprocal of each other. Which one is correct ?

(A) f'(1)=0

(B) $af'(1) < 0$

(C) $c \neq 0$

(D) $abc \neq 0$

Correct Option : B

SOLUTION

$$\alpha + \frac{1}{\alpha} > 2 \Rightarrow \frac{-b}{a} > 2 \Rightarrow \frac{2a+b}{a} < 0$$
$$\Rightarrow a(2a+b) < 0 \Rightarrow af'(1) < 0$$

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