JEE ADVANCED-PART TEST-8 (MATHS)

SOLUTION OF MOCK TEST

CONDUCTED ON DOUBTNUT APP

Q-1 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

There are 50 machines in a factory each productng 1000 bolts daily. For each additional machine installed, the output per machine drops by 10 bolts. How may additonal machines should be installed to maximize the total output per day?

(A) 20

(B) 30

(C) 50

(D) 25

Correct Option : D

SOLUTION

Let number of additional machines instanlled $= x$

$P(x) = (50 + x)(1000 - 10x) \Rightarrow P'(x) = 0 \Rightarrow x = 25$

Q-2 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

If $a^2x^4 + b^2y^4 = c^6$ then the maximum value of xy is $(a, b, c > 0)$

(A) $\dfrac{c^2}{\sqrt{ab}}$

(B) $\dfrac{c^3}{ab}$

(C) $\dfrac{c^3}{\sqrt{2ab}}$

(D) $\dfrac{c^3}{2ab}$

Correct Option : C

SOLUTION

Let $\frac{ax^2}{c^3} = \sin\theta$ and

$\frac{by^2}{c^3} = \cos\theta \Rightarrow xy = \sqrt{\frac{c^6 \sin\theta\cos\theta}{ab}} \Rightarrow (xy)_{\max} = \sqrt{\frac{c^6}{2ab}}$

Q-3 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

The point on the curve $xy^2 = 1$ nearest to origin is

(A) $\left(2^{-1/3}, \pm 2^{1/6}\right)$

(B) $\left(2^{-1/3}, 2^{-1/6}\right)$

(C) $\left(2^{1/3}, \pm^{1/6}\right)$

(D) $(1, 1)$

Correct Option : A

SOLUTION

$$S=\sqrt{x^2+y^2}=\sqrt{x^2+\frac{1}{x}}\Rightarrow\frac{dS}{dx}=0\Rightarrow x=\left(\frac{1}{2}\right)^{1/3}$$

Q-4 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

The fraction exceeding its own n^{th} power $(n \in N)$ by maximum possible value is

(A) $\left(\frac{1}{n}\right)^{\frac{1}{n-1}}$

(B) $\left(\frac{1}{n}\right)^{n-1}$

(C) $\left(\frac{1}{n}\right)^{n}$

(D) $\left(\frac{1}{n}+1\right)^{\frac{1}{n-1}}$

Correct Option : A

SOLUTION

Let x be the fractiion

$\Rightarrow y = x - x^n$ has to be maximum

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow x = \left(\frac{1}{n}\right)^{\frac{1}{n-1}} \& \frac{d^2y}{dx^2} < 0$$

Q-5 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

A person is standing at the edge of a slow moving river which is 1 km wide. He wishes to return to the camp ground on the opposite side of the river for which he may swim to any point on the opposite bank and then walk for the distance. The campground is 1 km away from the point on the opposite bank directly across from where he starts to swim. if he swims at the rate of 2 km/hr and walks at the rate of 3 km/hr the minimum time taken by him is approximately

(A) 0.6 hr

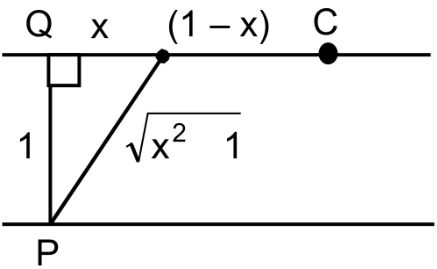
(B) 0.7 hr

(C) 0.8 hr

(D) 0.9 hr

Correct Option : B

SOLUTION



$$T = D/S \Rightarrow T = \frac{\sqrt{x^2+1}}{2} + \frac{1-x}{3} = f(x) \text{ (say)}$$

$$f'(x) = 0 \Rightarrow x = \frac{2}{\sqrt{5}} \text{ for minima}$$

$$\Rightarrow f(x) = \text{ minimum line } = 0.7 \text{ approximately}$$

Q-6 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

The greatest value of the

$$f(x) = \left(3 - \sqrt{4 - x^2}\right)^2 + \left(1 + \sqrt{4 - x^2}\right)^3 \text{ is}$$

(A) 25

(B) 26

(C) 28

(D) 24

Correct Option : C

SOLUTION

$$\sqrt{4 - x^2} = t$$

$$f(t) = (3 - 1)^2 + (1 + t)^3$$

$$f'(t) = 0 \Rightarrow t = -3 \text{ (rejected) } 1/3$$

$\therefore$ maximum occurs at end point

$$f_t(0) = 10 \& f_t(2) = 28 \Rightarrow [f(x)]_{\max} = 28$$

Let f(x) be a non-negative continuous function satisfying

$f'(x)\cos x \leq f(x)\sin x \,\forall x \geq 0$ then

$f\left(\frac{5\pi}{3}\right) =$

(A) $e^{-1/2}$

(B) $\frac{1}{\sqrt{2}}$

(C) 0

(D) $\frac{1}{2}$

Correct Option : C

SOLUTION

Let $g(x) = f(x)\cos x \Rightarrow g'(x) \leq 0 \Rightarrow g\left(\frac{5\pi}{3}\right) \leq g\left(\frac{\pi}{2}\right)$

$\Rightarrow g\left(\frac{5\pi}{3}\right) \leq 0 \Rightarrow \frac{1}{2}f\left(\frac{5\pi}{3}\right) \leq 0 \Rightarrow f\left(\frac{5\pi}{3}\right) = 0$

if $f(x)=\begin{cases}-\cos^2\left(\frac{\pi X}{2}\right) & 0\le x<1\\ (1-x)^2 & 1\le x\le 2\end{cases}$ Then number of values of obtained by applying LMVT on $f(x)$ in interval [0,2] is

(A) 1

(B) 2

(C) 3

(D) LMVT is not applicable

Correct Option : C

SOLUTION

$$f'(x)=\frac{f(2)-f(0)}{2-0}\Rightarrow f'(c)=1\Rightarrow \frac{\pi}{2}\sin(\pi C)=1,\ -2(1-c)$$

$$\Rightarrow \sin\pi c=\frac{2}{\pi}, c=\frac{3}{2}\Rightarrow \text{ three values of c}$$

Q-9 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Tangent lines are drawn at the point of inflexion of the function $f(x) = \cos x$ on $[0, 2\pi]$. The lines intersect with the x-axis so as to form a triangle. The area of this triangle is

(A) $\frac{\pi^2}{2}$

(B) $\frac{\pi^2}{4}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{16}$

Correct Option : B

SOLUTION

Infection points are $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Tangents are $y = -x + \frac{\pi}{2}, y = x - \frac{3\pi}{2}$

Area of triangle $= \frac{1}{2} \times \frac{\pi}{2} \times \pi = \frac{\pi^2}{4}$

Q-10 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Let f(x) be a twice differentiable positive function on an interval (a,b). Define a function g(x) such that $f(x) = e^{g(x)} \forall x \in (1, b)$. If $f(x)$ is such that the roots of the equation $f(x)t^2 - [f(x)]t + f''(x) = 0$ are always real and distinct, then

(A) g(x) is always increasing on (a,b)

(B) g(x) is always decreasing

(C) g(x) is always concave up on (a,b)

(D) g(x) is always concave downward on (a,b)

Correct Option : D

SOLUTION

$g(x) = \ln[f(x)]$

$$g''(x) = \frac{f(x)f''(x) - [f'(x)]^2}{[f(x)]^2} < 0 \because D > 0 \text{ for quadratic}$$

Q-11 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

A lamp of negligible height is placed on the ground at a distance of 10 from a wall. A man 2m tall is walking at a speed of 1m/s from the lamp towards the wall. When he is 5 m away from the wall, the rate at which his shadow shortens on the wall is

(A) $\frac{3}{2}m/s$

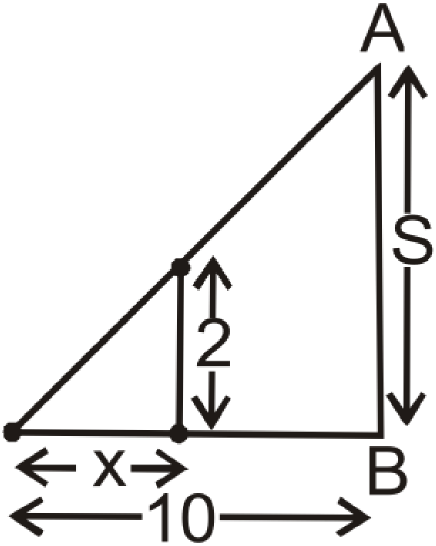
(B) $\frac{3}{5}m/s$

(C) $\left(\frac{4}{5}\right)m/s$

(D) $1m/s$

Correct Option : C

SOLUTION



$$\frac{S}{10} = \frac{2}{x} \Rightarrow \frac{ds}{dt} = \frac{-20}{x^2}\frac{dx}{dt}$$

$$x = 5 \Rightarrow \frac{ds}{dt} = \frac{-20}{25} \times 1 = -\frac{4}{5}m/s$$

Q-12 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

The point of intersection of tangent to the curve $y = x^4$ at $(1, 1)$ with the normal to the same curve at $(-1, 1)$ is

(A) $\left(\frac{17}{15}, \frac{13}{15}\right)$

(B) $\left(\frac{7}{5}, \frac{3}{5}\right)$

(C) $\left(\frac{17}{15}, \frac{23}{15}\right)$

(D) does not exist

Correct Option : C

SOLUTION

$y = x^4$

$\therefore \frac{dy}{dx} = 4x^3 \therefore \frac{dy}{dx}|_{at\,(1,1)} = 4$

$\therefore$ Equation of tangent at (1, 1) is

$y - 1 = 4(x - 1) i.e 4x - y = 3$ ..(i)

$\frac{dy}{dx}|_{at\,(-1,1)} = -4$

$\therefore$ Slope of normal is $\frac{1}{4}$

$\therefore$ equation of the normal is $y - 1 = \frac{1}{4}(x + 1)$

i.e., $x - 4y = -5$

$\therefore$ point of intersection is $\left(\frac{17}{15}, \frac{23}{15}\right)$

Q-13 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

The tangent and normal at the point $P\left(at^2, 2\text{at}\right)$ to the parabola $y^2 = 4ax$ meet the x-axis in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent P to the circle through P, T, G is

(A) $\tan^{-1}\left(t^2\right)$

(B) $\cot^{-1}\left(t^2\right)$

(C) $\tan^{-1}(t)$

(D) $\cot^{-1}(t)$

Correct Option : C

SOLUTION

Equation of tangent and normal at $P\left(at^2, 2at\right)$ to the parabola is

$ty = x + at^2$ and $tx + y = 2at + at^3$

Putting $y=0$ in both we get $T\left(-at^2, 0\right), G\left(2a+at^2, 0\right)$

Since PT is $\perp$ to PG hence TG is diameter of circle through P,T,G

Hence the equation of circle

$\left(x+at^2\right)\left(x-2a-at^2\right)+(y-0)(y-0)=0$

$x^2+y^2-2ax-at^2\left(2a+at^2\right)=0$

$\frac{dy}{dx}=\frac{a-x}{y} \Rightarrow \frac{dy}{dx}|_P=\frac{1-t^2}{2t}$

and form $y^2=4ax$ we get $\frac{dy}{dx}|_P=\frac{1}{t}$

Q-14 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

If garphs of $y=\log_a x$ and $y=a^x (a>1)$ intersect at exactly one point then $a=$

(A) e

(B) $\sqrt{e}$

(C) e^e

(D) $e^{1/8}$

Correct Option : D

SOLUTION

$$\frac{d}{dx}(\log_a x) = \frac{d}{dx}(a^x) = 1 \Rightarrow a^x \log a = \frac{1}{x \log a} = 1$$

$$\Rightarrow x = \log_a e \text{ and } a^x = \frac{1}{\log a} \Rightarrow e = \frac{1}{\log a} \Rightarrow a = e^{1/e}$$

Q-15 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Minimum value of $(x_1 x_2)^2 + \left(\frac{x_1^2}{20} - \sqrt{(17 - x_2)(x_2 - 13)}\right)^2$

where $x_1 \in R^+$, $x_2 \in (13, 17)$ is

(A) $\left(5\sqrt{2} + 2\right)^2$

(B) $5\sqrt{2} + 2$

(C) $\left(5\sqrt{2} - 2\right)^2$

(D) $\left(5\sqrt{2}-2\right)$

Correct Option : C

SOLUTION

Given expression is square of shortest distance between curve

$x^2 = 20y$ and $(x-15)^2 + y^2 = 4$ since shortest distance between

two curves lies along their common normal.

Equation of normal at $(P(x_1, y_1)$ to $x^2 = 20y$ is

$y - \frac{x_1^2}{20} = \frac{10}{x_1}(x - x_1) \Rightarrow x_1 = 10 \Rightarrow y_1 = 5$

Hence shortest value of expression $= \sqrt{(10-15)^2 + 5^2} - 2$

Hence maximum value of expression $= \left(5\sqrt{2}-2\right)^2$

Q-16 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Let f(x) be a differentiable real valued of function satisfying

$f''(x) - 6f'(x) > 6 \forall x \leq 0$. If $f'(0) = -1$ ad $g(x) = f(x) + x$

then g(x) is

(A) increasig $\forall x \leq 0$

(B) decreasing $\forall x \leq 0$

(C) a constant function $\forall x \leq 0$

(D) None of these

Correct Option : A

SOLUTION

$f''(x) - 6f'(x) > 6 \Rightarrow e^{6x}(f''(x) - 6f'(x)) > 6e^{-6x}$

$\Rightarrow \frac{d}{dx}\left(e^{-6x}f'(x) + e^{-6x}\right) > 0 \Rightarrow e^{-6x}(f'(x) + 1)$ is increasing

also $h(x) = e^{-6x}[f'(x) + 1] = 0$ at $x = 0$

for $x \geq 0 \Rightarrow h(x) \geq h(0)$

$\Rightarrow e^{-6x}(f'(x) + 1) \geq 0$

$\Rightarrow f'(x) + 1 \geq 0 \forall x \in [0, \infty]$

$\therefore g'(x) = f'(x) + 1 \geq \ \forall x \in [0, \infty]$

$\because g(x)$ is increasing for $x \geq 0$

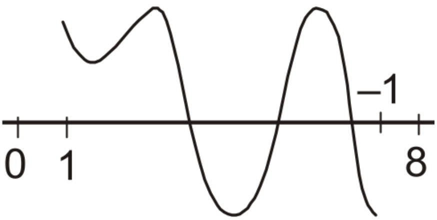
Q-17 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Let f(x) be a differentiable function on [0,8] such that

$f(1) = 3, f(2) = 1/2, f(3) = 4, f(4) = -2, f(5) = 6, f(6) = 1/$

thent he minimum number of points of intersection of the curves

$y = f'(x)$ and $y = f'(x)[f(x)]^2$ is



(A) 10

(B) 6

(C) 11

(D) 25

Correct Option : C

SOLUTION

$f'(x) = f'(x)f^2(x)$

$\Rightarrow$ either $f'(x) = 0$ at least 4

$\Rightarrow f'(x) = 1$ at least 5 root

$\Rightarrow f(x) = -1$ at least 2 root

$\Rightarrow$ a least 11 roots

Q-18 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Let $x^{2/3} + y^{2/3} = a^{2/3}$ be a equation of curve, then which of the following is correct?

(A) Equation of tangent at $P(x_1, y_1)$ to the curve is $\frac{x}{x_1^{1/3}} + \frac{y}{y_1^{1/3}} = a^{2/3}$

(B) Letngth of portion of tangent intercepted between coordinate axes is constant

(C) Equation of normal at $P(x_1, y_1)$ to the curve is

$$x x_1^{1/3} + y y_1^{1/3} = x_1^{4/3} + y_1^{4/3}$$

(D) if tangent at (x_1, y_1) meet the coordinate axes in A

and B, then locus of mid point of AB is

$$x^2 + y^2 = \frac{a^2}{4}$$

Correct Option : A

SOLUTION

Given curve $x^{2/3} + y^{2/3} = a^{2/3}$

slope of tangent $= \left\{\frac{dy}{dx}\right\}_{x_1, y_1} = -\frac{y_1^{1/3}}{x_1^{1/3}}$

Slope of thermal $= \frac{x_1^{1/3}}{y_1^{1/3}}$

Hence equation of tangent

$$y - y_1 = -\frac{y_1^{1/3}}{x_1^{1/3}}(x - x_1) \Rightarrow \frac{x}{x_1^{1/3}} + \frac{y}{y_1^{1/3}} = a^{2/3}$$

Now equation of normal

$$y - y_1 = \frac{x_1^{1/3}}{y_1^{1/3}}(x - x_1) \Rightarrow x x_1^{1/3} = x_1^{4/3} - y_1^{4/3}$$

Now tangent meet x-axis at $A\left(a^{2/3}x_1^{1/3}, 0\right)$ and y-axis at $B\left(0, a^{2/3}, y_1^{1/3}\right)$ and let mid point of AB is M(h,k)

Hence $h = \dfrac{a^{2/3}x_1^{1/3}}{2}, k = \dfrac{a^{2/3}y_1^{1/3}}{2}$

$x_1 = \left(\dfrac{2h}{a^{2/3}}\right)^3, y_1 = \left(\dfrac{2k}{a^{2/3}}\right)^3$

$x_1 = \dfrac{8h^3}{a^2}, y_1 = \dfrac{8k^3}{a^2}$

Now (x_1, y_1) satisfy given curve $x_1^{2/3} + y_1^{2/3} = a^{2/3}$

$\left(\dfrac{8h^3}{a^2}\right)^{2/3} + \left(\dfrac{8k^3}{a^2}\right)^{2/3} = a^{2/3}$ hence locus is $x^2 + y^2 = \dfrac{a^2}{4}$

Q-19 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Equation of tangent drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1,2) is/are

(A) $2\sqrt{3}(x-2) - y + 2\left(1+\sqrt{3}\right) = 0$

(B) $2\sqrt{3}(x-2) - y - 2\left(1+\sqrt{3}\right) = 0$

(C) $2\sqrt{3}(x-2) + y + 2\left(\sqrt{3}-1\right) = 0$

(D) $2\sqrt{3}(x-2)+y-2(\sqrt{3}-1)=0$

Correct Option : A

SOLUTION

Let tangent drawn from (1,2) meets the curve at (x_1, y_1)

Hence $\frac{dy}{dx}|_{x_1,y_1} = \frac{3x_1^2}{y_1-2} = \frac{y_1-2}{x_1-1}$

$\Rightarrow (y_1-2)^2 3x_1^3 - 3x_1^2$ ..(i)

(x_1, y_1) also satisfies $y^2 - 2x^3 - 4y + 8 = 0$

Hence $(y_1-2)^2 = 2x_1 - 4$ ..(ii)

from (i) & (ii) $3x_1^3 - 3x_1^2 = 2x_1^3 - 4$

$\Rightarrow x_1^3 - 3x_1^2 + 4 = 0$

$\Rightarrow x_1 = -1, 2$

When $x-(1) = -1$ y_1 is imaginary root

$x_1 = 2 \quad y_1 = 2 \pm 2\sqrt{3}$

Hence equation of tangent $at\left(2, 2+2\sqrt{3}\right), \left(2, 2-\sqrt{3}\right)$ are

$2\sqrt{3}(x-2)-y+2\left(\sqrt{3}+1\right)=0$

$2\sqrt{3}(x-2)+y+2\left(\sqrt{3}-1\right)=0$

Q-20 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Which of the following pairs of curves is orthogonal

(A) $y^2 = 4ax, y = e^{-\frac{x}{2a}}$

(B) $y^2 = 4ax, x^2 = 4ay$ at $(0, 0)$

(C) $xy = a^2, x^2 - y^2 = b^2$

(D) $y = ax, x^2 + y^2 = c^2$

Correct Option : A

SOLUTION

(A). $y^2 = 4ax$ and $y = e^{-\frac{x}{2a}}$

$y' = \frac{dy}{dx} = \frac{2a}{y}$ and $y' = -\frac{1}{2a}e^{-\frac{x}{2a}} = -\frac{y}{2a}$

Hence $m_1m_2 = -1$

(B). $y^2 = 4ax$ and $x^2 = 4ay$

$y' = \frac{2a}{y_1}$ not defined at $x = 0$

$y' \frac{x_1}{2a} = 0$ at $x = 0$

Hence curves are orthogonal at (0,0)

(C). $xy = a^2$ and $x^2 - y^2 = b^2$

$x' = \frac{a^2}{x^2} \quad y' = \frac{x}{y}$

$m_1 m_2 = -\frac{a^2}{xy} = -1$

(D). $y = ax$ and $x^2 + y^2 = c^2$

$y' = a$ and $y' = -\frac{x}{y}$

$m_1 m_2 = -\frac{ax}{y} = -1$ are orthogonal

Q-21 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

If parametric equation of any curve at θ is given by

$x = a\cos^3\theta, y = a\sin^3\theta,$ then a $a > 0, \theta \in \left(0, \frac{\pi}{2}\right)$

(A) length of tangent $a\sin^2\theta$

(B) length of subtangent $a\sin^2\theta\cos\theta$

(C) length of normal $a\tan\theta\sin^2\theta$

(D) length of subnormal $a\sin^3\tan\theta$

Correct Option : A

SOLUTION

Given that $x = a\cos^2\theta, y = a\sin^3\theta$

$$\frac{dy}{dx} = \frac{dy / dth\eta}{dx / dth\eta} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

Length of tangent

$$= \left|\frac{y}{m}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right| = \frac{a\sin^3\theta\sqrt{1+\tan^2\theta}}{\tan\theta} = a\sin^2\theta$$

Length of subtangent $= \left|\frac{y}{m}\right| = \frac{a\sin^3\theta}{\tan\theta} = a\sin^2\theta\cos\theta$

Length of sub-normal $= |my| = a\sin^2\theta.\tan\theta$

Q-22 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Tangent at point P_1 (other than origin) on the curve $y = x^3$ meets the curve again at P_2 the tangent at P_2 meets the curve at P_3 and so on then

(A) abscissae of P_1, P_2, P_3 � P_n form a G.P

(B) ordinates of P_1, P_2, P_3 � . P_n form a G.P

(C) ratio of area of $\triangle P_1P_2P_3$ and $\triangle P_2P_3P_4$ is $\frac{1}{16}$

(D) ratio of area of $\triangle P_1P_2P_3$ and $\triangle P_2P_3P_4$ is 16

Correct Option : A

SOLUTION

Let any point of P_1 on $y = x^3$ (be (h, h^3)

tangent at P_1 is $y - h^3 = 3h^2(x - h)$

it meets $y = x^3$ at P_2

Hence $x^3 - h^3 = 3h^2(x - h)$

$x^2 + hx - 2h^2 = 0$

$x - h, \ - 2h$

Hence $P_2\left(- 2h, 8h^3\right)$

Tangent at $P_2\left(- 2h, \ - 8h^3\right)$ is $y + 8h^3 = 3(2h)^2(x + 2h)$

it meets $y = x^3$ at P_3 hence $x^3 + 8h^3 = 12h^2(x + 2h)$

or $x^2 - 2hx - 8h^2 = 0$

$x = 4h, \ - 2h$ continuing like this we get $P_4\left(- 8h, \ - 512h^3\right)$ and

so on

Q-23 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

$f(x) = 2e^x + \left(a^2 - 5a + 6\right)e^{-x} + \left(10 - 2a^2 - 11\right)x - 3$ is

increasing for all real value of x if $a \in$

(A) $\{2\}$

(B) [2,3]

(C) (2,3)

(D) $(3, \infty)$

Correct Option : A

SOLUTION

$$f'(x) \geq 0 \Rightarrow 2e^{x} - (a^2 - 5a + 6)e^{-x} + 10a - 2a^2 - 11 \geq 0$$

$$\Rightarrow e^{-x} + \left(5a - a^2 - \frac{11}{2}\right)e^{x} + (5a - a^2 - 6)\left(\frac{1}{2}\right) \geq 0$$

$$\Rightarrow (e^{x} + 5a - a^2 - 6)\left(e^{x} + \frac{1}{2}\right) \geq 0$$

$$a^2 - 5a + 6 \leq e^{x} \Rightarrow a^2 - 5a + 6 \leq 0 \Rightarrow 2 \leq a \leq 3$$

Q-24 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

Let f(x) be a function satisfring $f'(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$ and $f(0) = 0$ then

(A) $f(x) \geq 0 \forall x \in R$

(B) $f(x) \leq 0 \forall \xi n R$

(C) $f'(x)$ is increasing $\forall x \in R$

(D) $f(x)$ is even fucntion

Correct Option : A

SOLUTION

$f'(x) > 0 \forall x > 0 \& f'(x) \leq 0 \forall x < 0 \Rightarrow f(x) \geq 0 \forall x \in R \Rightarrow$

option (A) is true

$f''(x) = \dfrac{1}{\sqrt{x^2+1}} > 0 \Rightarrow f'(x)$ is increasing

Q-25 - JEE ADVANCED-PART TEST-8 (MATHS)-MATHS

$f(x) = \left(4\sin^2 x - 1\right)^n \left(x^2 + 6x + 11\right)$ where $n \in N$ then $x = \dfrac{\pi}{6}$ is a point of

(A) local maximum if n is even

(B) decreasing if n is odd

(C) local minimum if n is even

(D) increasing if n is odd

Correct Option : C

SOLUTION

$x^2 + 6x + 11 > 0 \quad \forall x \in R$

$$f\left(\frac{\pi}{6}\right) = 0, f\left(\frac{\pi^+}{6}\right) = (0^+)^n > 0, f\left(\frac{\pi^-}{6}\right) = (0^-)^n < 0$$