Q-1 - 12928203

Mass M is split into two parts m and $(M-m)$, which are then separated by a certain distance. What is the ratio of (m/M) which maximises the gravitational force between the parts ?

SOLUTION:

if r is the distance between m and $(M-m)$, the gravitational force between them will be

$$F = G\frac{m(M-m)}{r^2}$$

$$= \frac{G}{r^2}(mM - m^2)$$

For F to be maximum, $dF/dm = 0$ as M & r are constants.

$$\Rightarrow \frac{d}{dm}\left[\frac{G}{r^2}(mM - m^2)\right] = 0 \Rightarrow M - 2m = 0$$

$$\Rightarrow \frac{m}{M} = \frac{1}{2}$$

So, the force will be maximum when the parts are equal.

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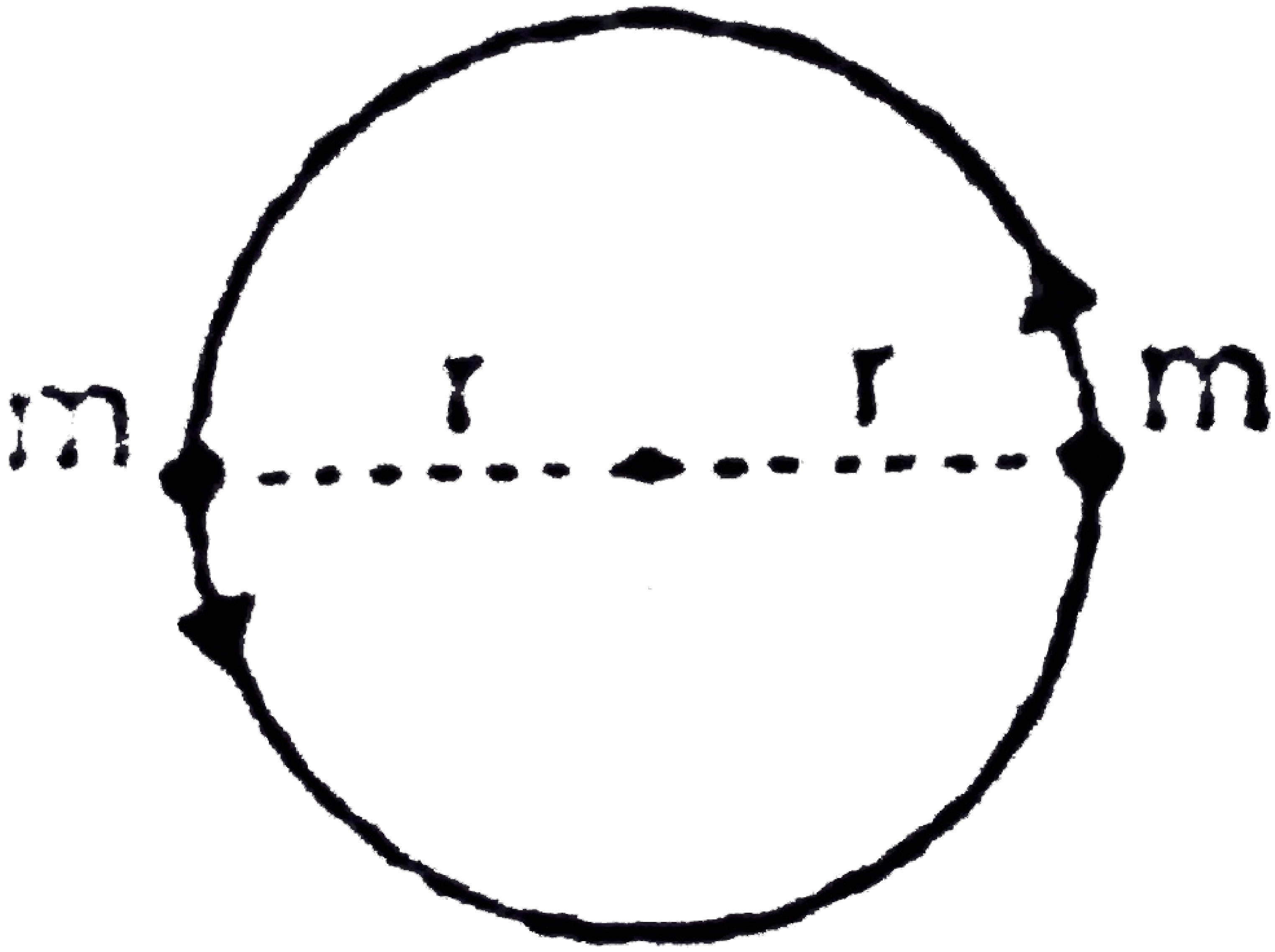
Q-2 - 14159491



Two particles of equal mass (m) each move in a circle of radius (r) under the action of their mutual gravitational attraction find the speed of each particle.

SOLUTION:

For motion of particle $\frac{mv^2}{r} = \frac{Gmm}{(2r)^2}$

$\Rightarrow v^2 = \frac{Gm}{4r} \Rightarrow v$
$= \frac{1}{2}\sqrt{\frac{Gm}{r}}$

Q-3 - 17240619

The masses and radii of the earth an moon are M_1 and R_1 and M_2, R_2 respectively. Their centres are at a distacne r apart. Find the minimum speed with which the particle of mass m should be projected from a point mid-way between the two centres so as to escape to infinity.

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Q-4 - 11302745

What is the mass of the planet that has a satellite whose time period is T and orbital radius is r?

(A) $\dfrac{4\pi^3 r^3}{GT^2}$

(B) $\dfrac{4\pi^2 r^3}{GT^2}$

(C) $\dfrac{4\pi^2 r^3}{GT^3}$

(D) $\dfrac{4\pi^2 T}{GT^2}$

CORRECT ANSWER: A

SOLUTION:

Suppose that a satellite of mass m describes a circular orbit around a planetof mass M.

$$F = \frac{GmM}{r^2}$$

This force must be mass times the cenripetal acceleration.

$$\therefore F = \frac{mv^2}{r} = m\omega^2$$

$$= \frac{4\pi^2}{T^2} r$$

$$\therefore M = \frac{4\pi^2 r^3}{GT^2}$$

Q-5 - 18247247

The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv_e, where v_e is its escape velocity from the surface of the earth. Find the value of f.

SOLUTION:

For a platform at a height h,

escape energy = binding energy of sphere

$$\Rightarrow \frac{1}{2}mv'^2_e = \frac{GMm}{R+h} \Rightarrow v'_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{2R}} \quad (\because h = R)$$

But at surface of earth, $v_e = \sqrt{\frac{2GM}{R}}$

As given, $v'_e = fv_e$

Hence,

$$\sqrt{\frac{2GM}{2R}} = f\sqrt{\frac{2GM}{R}}$$

$$\text{or } \frac{1}{2R} = \frac{f^2}{R} \Rightarrow f = \frac{1}{\sqrt{2}}$$

.

Q-6 - 12230280

Statement-1: Escape velocity is independent of the angle of projection.

Statement-2: Escape velocity from the surface of earth is $\sqrt{2gR}$ where R is radius of earth.

(A) Statement-1 is true, statement-2 is true, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, Statement-2 is true, statement-2 is Not a correct explanation for statement-1

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false Statement-2 is True.

CORRECT ANSWER: A

SOLUTION:

Using only energy conservation

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R} = 0$$

$$+ 0$$

.

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Q-7 - 14948324

A satellite of mass m is orbiting the earth in a cicular orbit of radius r. It starts losing its mechanical energy due to small air resistance at the rate of k joule/sec. The time taken by the satellite to hit the suface of the earth is (M and R are the mass and radius of the earth)

(A) $\frac{GMm}{K}\left(\frac{1}{R} - \frac{1}{r}\right)$

(B) $\frac{GMm}{2K}\left(\frac{1}{R} - \frac{1}{r}\right)$

(C) $\frac{2GMm}{K}\left(\frac{1}{R} - \frac{1}{r}\right)$

(D) $\frac{GMm}{K}\left(\frac{1}{R}+\frac{1}{r}\right)$

CORRECT ANSWER: B

SOLUTION:

$$F.\,v = k \Rightarrow m.\,\frac{dv}{dt}.\,v$$
$$= k$$

$$\int_0^t dt = \frac{m}{k}\int_{v_i}^{v_t} vdv$$
$$= \frac{m}{2k}\left(\frac{6M}{R}-\frac{6M}{r}\right)$$

$$t = \frac{6Mm}{2k}\left(\frac{1}{R}-\frac{1}{r}\right)$$

Q-8 - 15835939

If acceleration due to gravity on the surface of a planet is two times that on surface of earth and its radius is double that of earth. Then escape velocity from the surface of that planet in comparison to earth will be

(A) 2 v

(B) 3 v

(C) 4 v

(D) None of these

CORRECT ANSWER: A

Q-9 - 12928190

An artificial satellite is in an elliptical orbit around the earth with aphelion of $6R$ and perihelion of $2R$ where R is radius of the earth $= 6400 km$. Calculate the eccentricity of the elliptical orbit.

SOLUTION:

We know that,

perigee $(r_p) = a(1-e) = 2R$.....(1)

apogee $(r_a) = a(1+e) = 6R$.....(2)

Solving (1) & (2), eccentricity $(e) = 0.5$

Q-10 - 12928447

Gravitational force between two point masses m and M separated by a distance r is F. Now if a point mass $3m$ is placed very next to m, the total force on M will be

(A) F

(B) $2F$

(C) $3F$

(D) $4F$

CORRECT ANSWER: D

SOLUTION:

$$F = \frac{GMm}{r^2}, F' = \frac{GM(m+2m)}{r^2}$$

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Q-11 - 9527409

The time take Mars to revolve round the sun is 1.88 years.Find the ratio of average distance between mars and the sun to that between the earth and the sun.

CORRECT ANSWER: A::B

SOLUTION:

According to Keler's laws of planetary motion

$T^2 \alpha R^3$

$$\rightarrow \frac{T_m^2}{T_3^2} = \frac{R_{ms}^3}{R_{es}^3}$$

$$\rightarrow \frac{Rms}{Rm} = \frac{1.88}{1}$$

$$\therefore \frac{Rm}{R_{es}} = (1.88)^{\frac{2}{3}}$$

$$= 1.52$$

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Q-12 - 10058826

A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is hours.

CORRECT ANSWER: D

SOLUTION:

According to kepler 's law T^2 prop R^3(T_1^2)/(T_2^2) = (R_1^3)/(R_2^3) Here R_1 = R+6R = 7R and R_2 = 2.5 R+R = 3.5RrArr (24xx24)/(T_2^2) = (7xx7xx7xxR^3)/(3.5xx3.5xx3.5xxR^3) rArrr T_2 = 8.48 hr`

Q-13 - 11748572

The orbit of geostationary satellite is circular, the time period of satellite depeds on (i) mass of the satellite, (ii) mass of earth, (iii) readius of the orbit and (iv) height of the satellite from the surface of the earth

(A) (i) only

(B) (i) and (ii)

(C) (i), (ii) and (iii)

(D) (ii), (iii) and (iv)

CORRECT ANSWER: D

SOLUTION:

Orbital velocity, $v_0 = \sqrt{\frac{GM_E}{R_E + h}}$

Time period,

$$T = \frac{2\pi(R_E + h)}{v_0}$$

$$= \frac{2\pi(R_E + h)^{3/2}}{(GM_E)^{1/2}}$$

Thus, the time period of satellite in independent of mass of satellite but depends on mass of the earth, radius of the orbit $(R_E + h)$, height of the mass satellite from the surface of the earth.

Q-14 - 15085323

The height of geostationary orbit above the surface of the earth is h. Radius of the earth is R. The earth shrinks to half its present radius (mass remaining unchanged). Now hat will be height of a geostationary satellite above the surface of the earth?

CORRECT ANSWER: $H+\frac{R}{2}$

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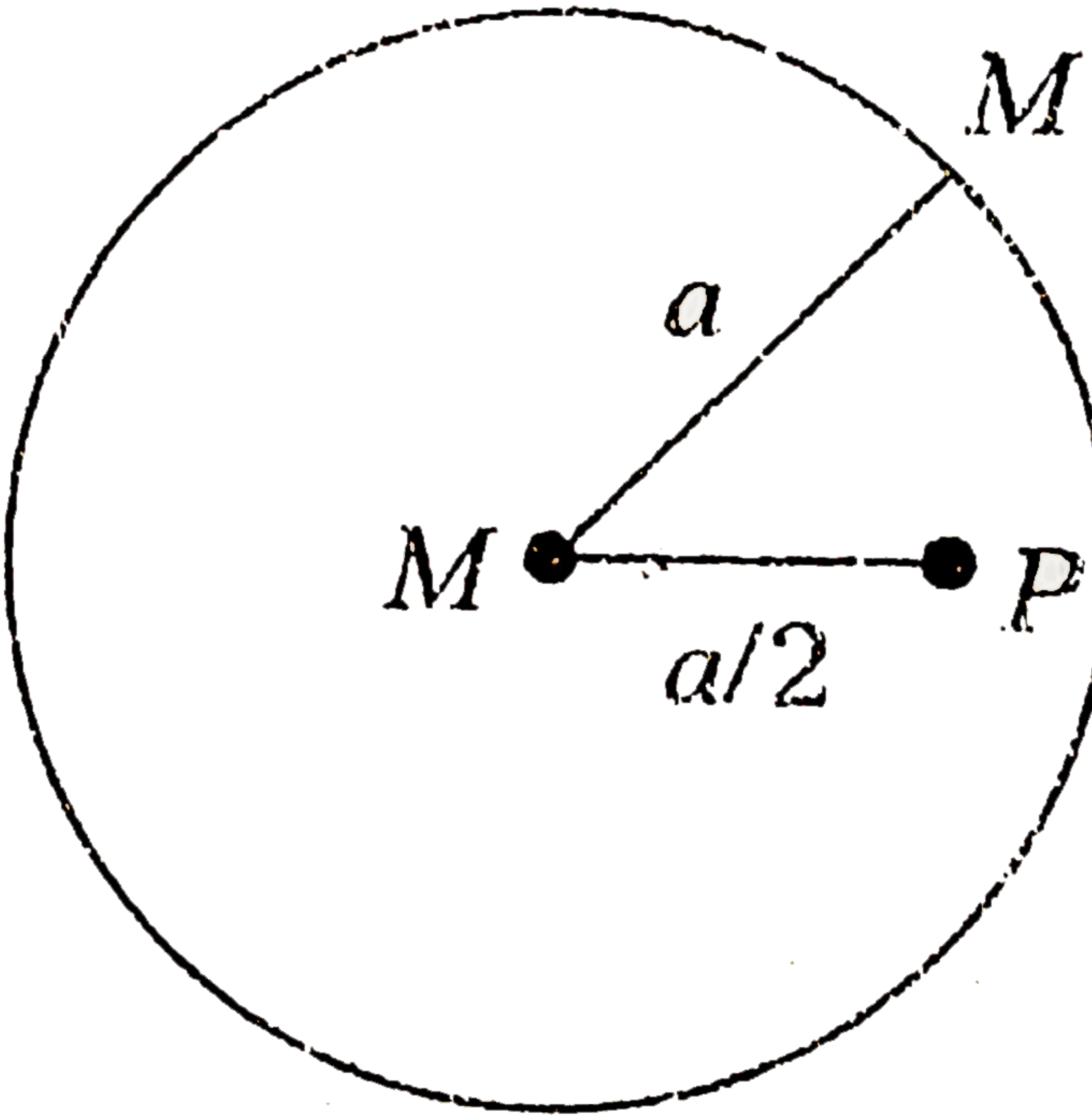
Q-15 - 18247226

A particle of mass M is placed at the centre of a spherical shell of same mass and radius a. What will be the magnitude of the gravitational potential at a point situated at a/2 distance from the centre ?

SOLUTION:

The situation given in the question can be pictutised as shown below



Therefore, the magnitudee of gravitational potential at a poinnt situated at a/2 distance from the centre can be given as

$$V_P = V_{\text{sphere}} + V_{\text{particle}}$$
$$= \frac{-GM}{a} - \frac{GM}{a/2}$$
$$= \frac{-3GM}{a}$$

.

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Q-16 - 13074097

A planet is moving in an elliptic orbit. If T, V, E and L stand, respectively, for its kinetic energy, gravitational potential energy, total energy and angular momentum about the centre of force, then

(A) T is conserved

(B) V is always positive

(C) E is always negative

(D) magnitude of L is conserved but its direction changes continuously

---

CORRECT ANSWER: C

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Q-17 - 12230291

Statement-1:The escape speed of a body of mass m is v_e. The escape speed of another body of mass $2m$ for same planet is v_e.

Statement-2: The escape speed of a body for a given planet is independent of mass of body.

(A) Statement-1 is true, statement-2 is true, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is true, Statement-2 is true, statement-2 is Not a correct explanation for statement-1

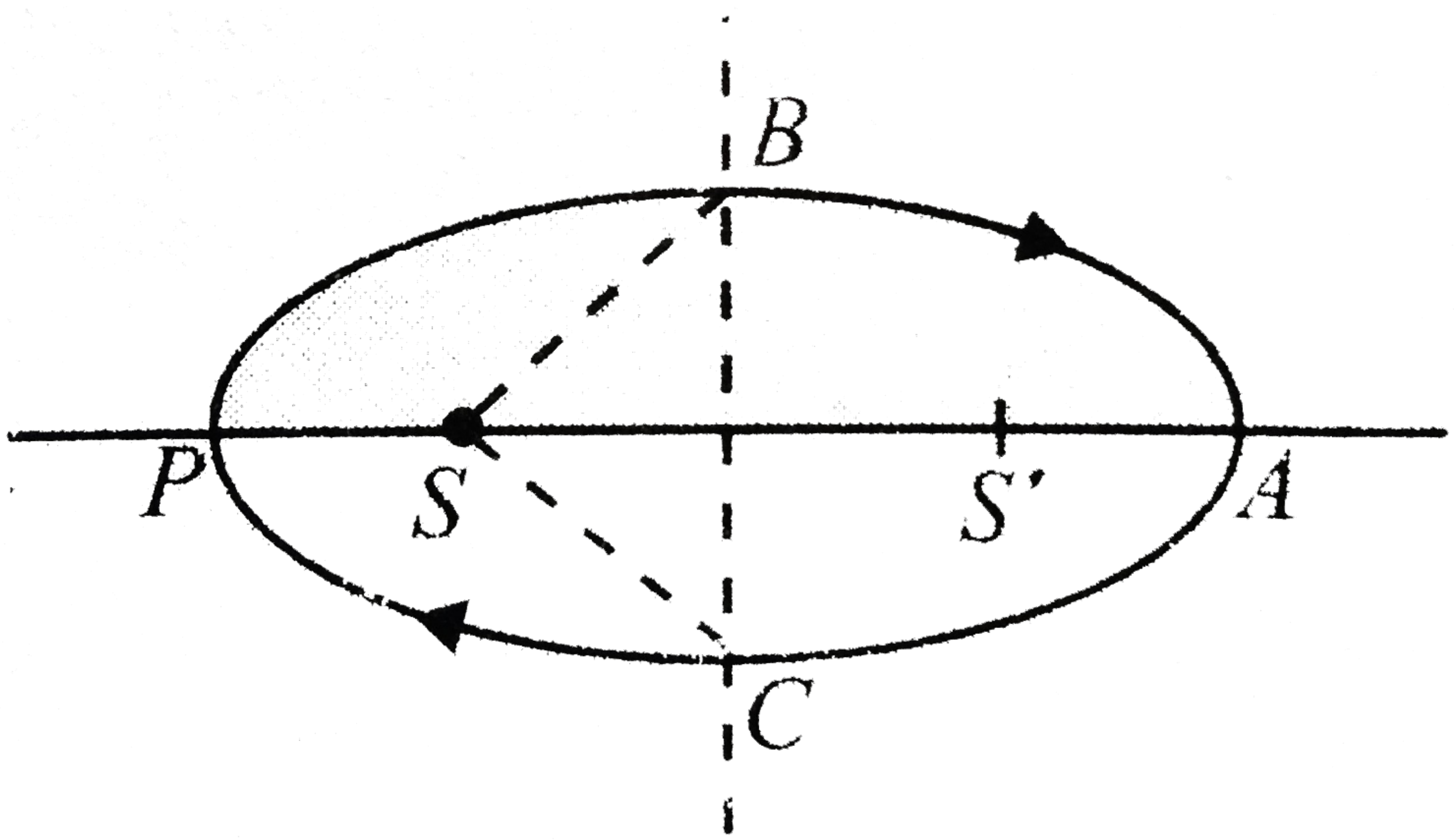
(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false Statement-2 is True.

CORRECT ANSWER: A

Q-18 - 11302549

Let the speed of the planet at the perihelion P in figure be v_P and the Sun planet distance SP be r_P. Relater r_P, v_P to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to transverse BAC and CPB?



SOLUTION:

Referring fig we note that $\overrightarrow{r}_P$ and $\overrightarrow{v}_P$ are perpendicular to each other. Similarly $\overrightarrow{r}_A$ and $\overrightarrow{v}_A$ are perpendicular to each. Using the law of conservation of angular momentum,

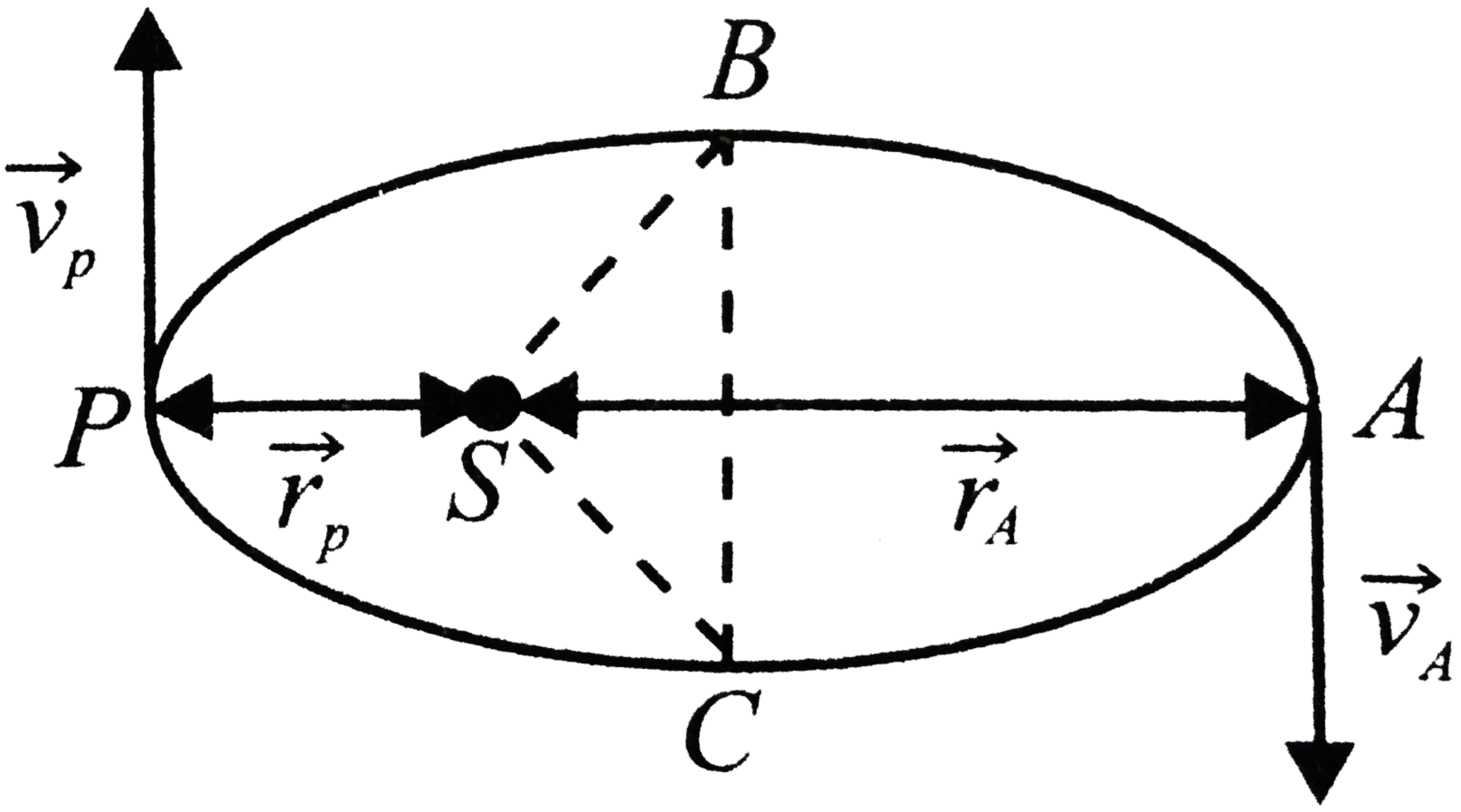
Angular momentum of the plane at $P =$ Angular momentum of the planet at A

$$\Rightarrow m_P v_P r_P$$
$$= m_A v_A r_A$$

or $\dfrac{v_P}{v_A} = \dfrac{r_A}{r_P}$

Since $r_A > r_P, v_P > v_A$

Her area $SBAC$ is greater than area $SCPB$. As the areal velocity of a planet is contant aroung the Sun i.e. equal areas are swept in equal times. Hence the planet will take longer time to reverse BAC than CPB.



Q-19 - 18247415

An earth satellite of mass m revolves in a circular orbit at a height h from the surface of the earth. R is the radius of the earth and g is acceleration due to gravity at the surface of the earth. The velocity of the satellite in the orbit is given by

(A) $\frac{gR^2}{R+h}$

(B) gR

(C) $\frac{gR}{R+h}$

(D) $\sqrt{\dfrac{gR^2}{R+h}}$

CORRECT ANSWER: D

SOLUTION:

Orbital velocity,

$$v_o = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{gR^2}{R+h}}$$

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Q-20 - 11748682

Acceleration due to gravity on moon is $1/6$ of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon (R_m) in terms of R_e will be

(A) $\frac{5}{18}R_e$

(B) $\frac{1}{6}R_e$

(C) $\frac{3}{18}R_e$

(D) $/\left(2\sqrt{3}\right)R_e$

CORRECT ANSWER: A

SOLUTION:

$$g = \frac{4}{3}\pi G\rho R \Rightarrow g$$

$$\propto \rho R \Rightarrow \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m}$$

$$\times \frac{R_e}{R_m}$$

$$\Rightarrow \frac{6}{1} = \frac{5}{3} \times \frac{R_e}{R_m}$$

$$\Rightarrow R_m = \frac{5}{18}R_e$$

Q-21 - 11748631

Two sphere of masses m and M are situated in air and the gravitational force between them is F. The space around the masses in now filled with a liquid of specific gravity 3. The gravitational force will now be

(A) $\frac{F}{9}$

(B) $3F$

(C) F

(D) $\frac{F}{3}$

CORRECT ANSWER: C

SOLUTION:

Gravitational force is independent of the medium.

Hence, this will remain the same.

Q-22 - 18247459

Assertion : Gravitational force between two masses in air is F. If they are immersed in water, force will remain F

Reason : Gravitational force does not depend on the medium between the masses.

(A) If both Assertin and Reason are correct and Reason is the correct explanation of Assertion

(B) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion

(C) If Assertion is true but Reason is false

(D) If Assertion is false but Reason is true

CORRECT ANSWER: A

Q-23 - 12928443

A planet moves around the sun. at a given point P, it is closest from the sun at a distance d_1, and has a speed V_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

(A) $\dfrac{d_1^2 V_1}{d_2}$

(B) $\dfrac{d_2 V_1}{d_1}$

(C) $\dfrac{d_1 V_1}{d_2}$

(D) $\dfrac{d_2^2 V_1}{d_1^2}$

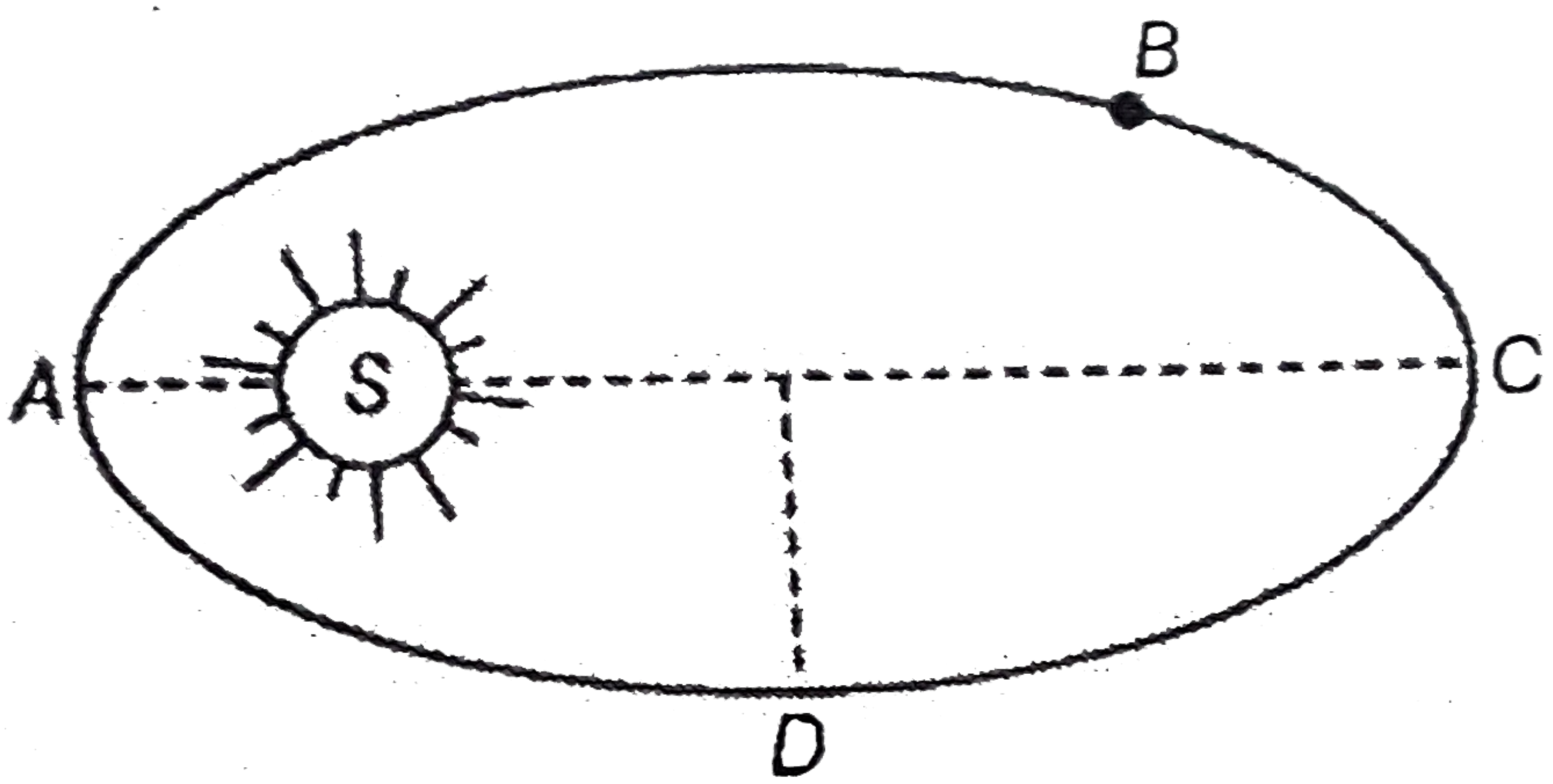
CORRECT ANSWER: C

SOLUTION:

From conservation of angular momentum vr=constant.

Q-24 - 18254034

A planet revolves in elliptical orbit around the sun. (see figure). The linear speed of the planet will be maximum at



(A) A

(B) B

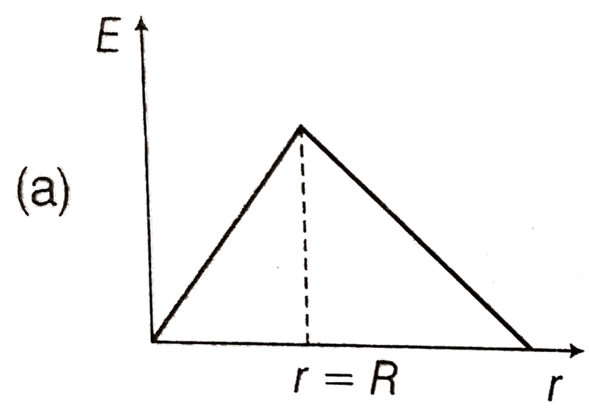
(C) C

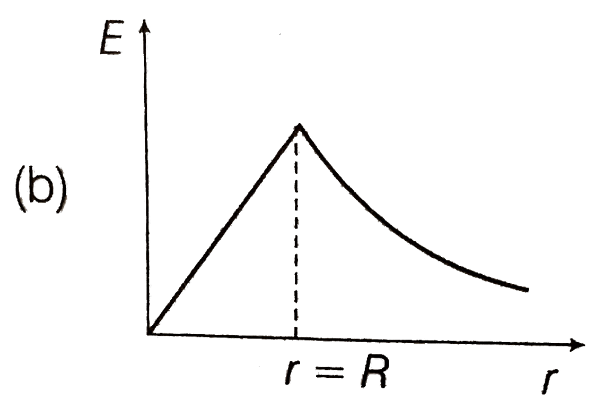
(D) D

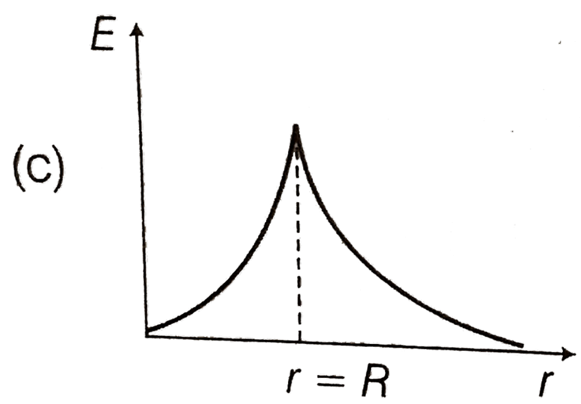
CORRECT ANSWER: A

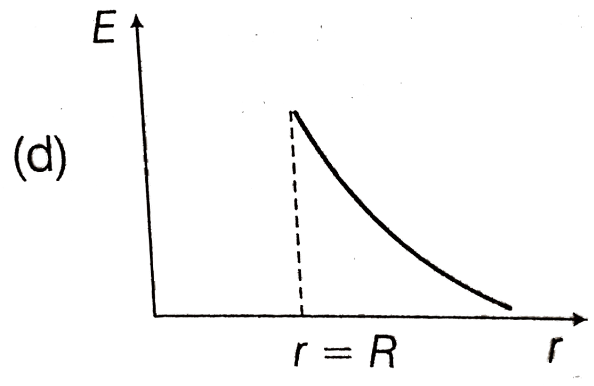
Q-25 - 18247309

Which one of the following graphs represents correctly represent the variation of the gravitational field (E) with the distance (r) from the centre of a spherical shell of mass M radius R ?

(A) 

(B) 

(C) 

(D) 

CORRECT ANSWER: D

SOLUTION:

Intensity will be zero inside the spherical shell.

E= 0 upto r = R and $E \propto \frac{1}{r^2}$ when $r > R$.

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Q-26 - 11302758

A body of mass m rises to a height $h = R/5$ from the earth's surface where R is earth's radius. If g is acceleration due to gravity at the earth's surface, the increase in potential energy is

(A) mgh

(B) $\frac{4}{5} mgh$

(C) $\frac{5}{6} mgh$

(D) $\frac{6}{7} mgh$

CORRECT ANSWER: C

SOLUTION:

Increase is gravitational potential energy is

$$\left[\frac{GMm}{R+\frac{R}{5}}\right]\left[-\frac{GMm}{R}\right]$$

$$=\frac{GMm}{R}$$

$$=\frac{GMm \times 5}{6R}$$

$$=\frac{GMm}{R}\left[1-\frac{5}{6}\right]$$

$$=\frac{GMm}{R} \times \frac{1}{6}$$

$$= \frac{mgR^2}{6R} = \frac{1}{6}$$

$$+ mgR = \frac{5}{6} mgh[$$

$$\because R = 5h]$$

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Q-27 - 12928359

Two artificial satellites are revolving in the same circular orbit.

Then they must have the same

(A) mass

(B) angular momentum

(C) kinetic energy

(D) period of revolution

CORRECT ANSWER: D

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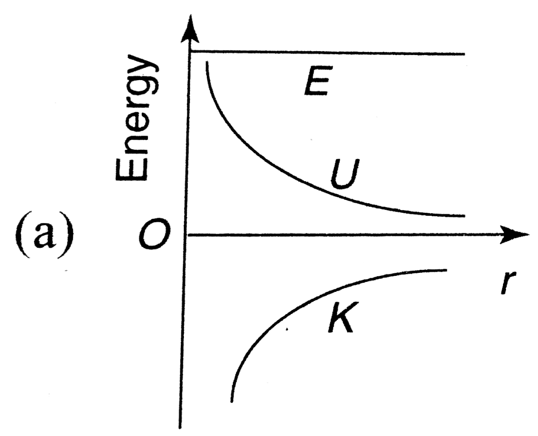
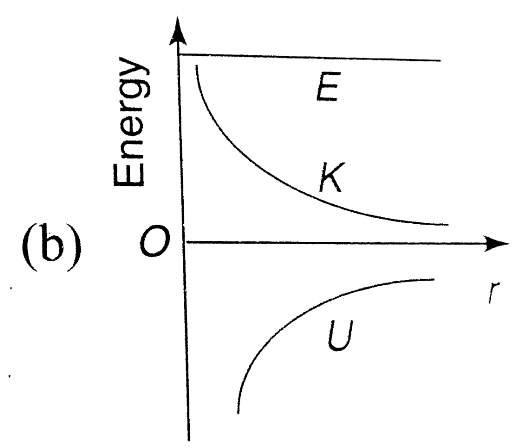
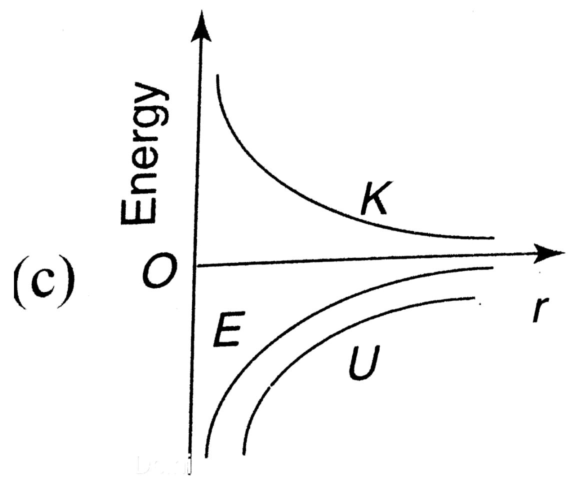
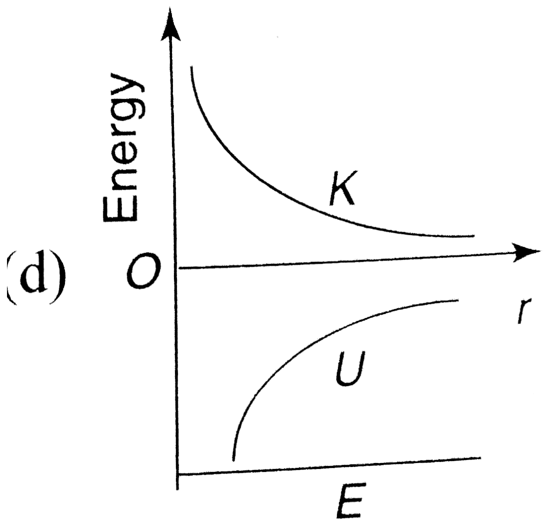
Q-28 - 11748540

The correct graph representing the variation of total energy (E_t), kinetic energy (E_k) and potential energy (U) of a satellite with its distance form the centre of earth is

(A) 

(B) 

(C) 

(D) 

CORRECT ANSWER: C

SOLUTION:

$$U = -\frac{GMm}{r}, K$$
$$= \frac{GMm}{2r}$$
and $E = \frac{-GMm}{2r}$

For a satellite U, K and E very with r and also U and E remain negative whereas K remain always positive

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Q-29 - 12230250

A satellite is moveing in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5 J$. The amount of energy to be imparted to the satellite to transfer it to be orbit where its potential energy is $U = -2 \times 10^5 J$ is equal to ....

CORRECT ANSWER: $1XX10^5 J$

SOLUTION:

$$E_1 = \frac{U_1}{2} \Rightarrow E_2 = \frac{U_2}{2}$$
$$= - \times 10^5 J$$

$$E_1 = -2 \times 10^5 J$$

So
$$\Delta E = E_2 - E_1 = 1$$
$$\times 10^5 J$$

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Q-30 - 12928388

A satellite moving in a circular path of radius r around earth has a time period T. If its radius slightly increases by $4\ \%$, then percentage change in its time period is

(A) $1\ \%$

(B) $6\ \%$

(C) $3\ \%$

(D) $9\ \%$

CORRECT ANSWER: B

SOLUTION:

$$T^2 \alpha r^3, \frac{\Delta T}{T} \times 100$$
$$= \frac{3}{2} \frac{\Delta R}{R} \times 100$$

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Q-31 - 14948324

A satellite of mass m is orbiting the earth in a cicular orbit of radius r. It starts losing its mechanical energy due to small air resistance at the rate of k joule/sec. The time taken by the satellite to hit the suface of the earth is (M and R are the mass and radius of the earth)

(A) $\frac{GMm}{K}\left(\frac{1}{R}-\frac{1}{r}\right)$

(B) $\frac{GMm}{2K}\left(\frac{1}{R}-\frac{1}{r}\right)$

(C) $\frac{2GMm}{K}\left(\frac{1}{R}-\frac{1}{r}\right)$

(D) $\frac{GMm}{K}\left(\frac{1}{R}+\frac{1}{r}\right)$

CORRECT ANSWER: B

SOLUTION:

$F.v=k \Rightarrow m.\frac{dv}{dt}.v$

$=k$

$$\int_0^t dt = \frac{m}{k}\int_{v_i}^{v_t} vdv$$

$$= \frac{m}{2k}\left(\frac{6M}{R} - \frac{6M}{r}\right)$$

$$t = \frac{6Mm}{2k}\left(\frac{1}{R} - \frac{1}{r}\right)$$

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Q-32 - 12007219

A rocket is fired vertically upwards with a speed of $v(=5kms^{-1})$ from the surface of earth. It goes up to a height h before returning to earth. At height h a body is thrown from the rocket with speed v_0 in such away so that the body becomes a satellite of earth. Let the mass of the earth, $M = 6 \times 10^{24} kg$, mean radius of the earth,

$R = 6.4 \times 10^6 m, G = 6.67 \times 10^{-11} Nm^2 kg^{-2}, g = 9.8 ms^{-2}$

.

Answer the following questions:

Time period of revollution of satellite around the earth is

(A) $3550s$

(B) $7100s$

(C) $5330s$

(D) $8880s$

CORRECT ANSWER: B

SOLUTION:

Time period of revolution of satellite,

$$T = \frac{2\pi(R+h)}{v_0}$$

$$= \frac{2 \times (22/7) \times (6.4 \times 10^6 + 1.6 \times 10^6)}{7.1 \times 10^3}$$

$= 7082 \approx 7100s$

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Q-33 - 11748573

A body is fired with a velocity of magnitude $\sqrt{gR} < V\sqrt{2gR}$ at an angle of 30 with the radius vector of earth. If at the highest point the speed of the body is $V/4$, the maximum height attained by the body is equal to:

(A) $V^2/8g$

(B) R

(C) $\sqrt{2}R$

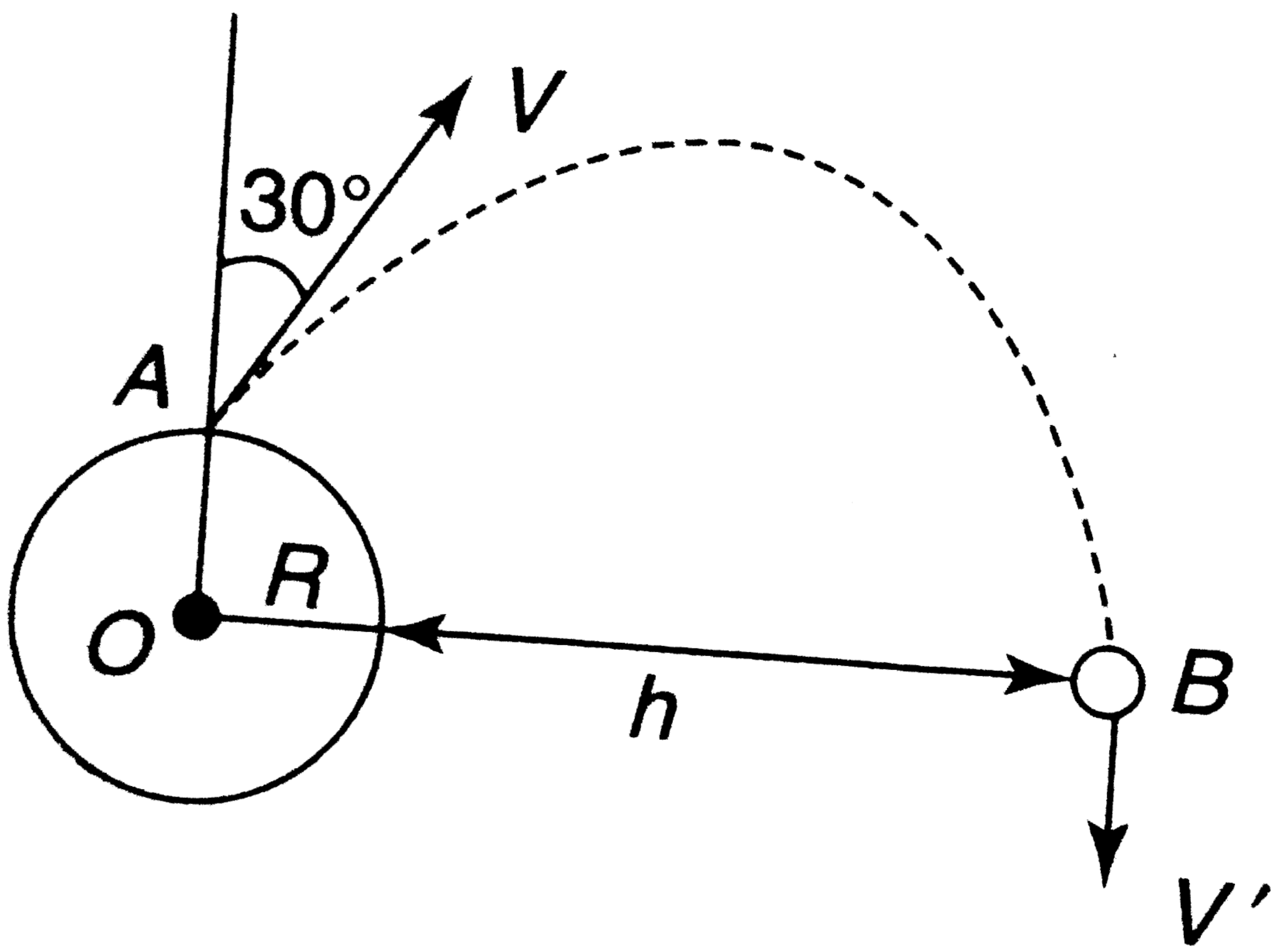
(D) none of these

CORRECT ANSWER: A

SOLUTION:

Coservation of angular momentum of the body about O

yields



$(mV \sin 30)R = mV$
$'(R+h)$

$\frac{V}{2}R = \frac{V}{4}(R+h)\Bigg[$
$\therefore V' = \frac{V}{4}\Bigg]$

$\therefore h = R$

Q-34 - 12230230

A planet revolves about the sun in elliptical orbit. The arial velocity $\left(\frac{dA}{dt}\right)$ of the planet is $4.0 \times 10^{16} m^2 / s$. The least distance between planet and the sun is $2 \times 10^{12} m$. Then the maximum speed of the planet in km / s is -

(A) 10

(B) 20

(C) 40

(D) None of these

CORRECT ANSWER: C

SOLUTION:

$$2m\frac{dA}{dt} = mvr$$

$$2 \times 4 \times 10^{16} = v_{\max} \times 2 \times 10^{12}$$

$$v_{\max} = 40km / s$$

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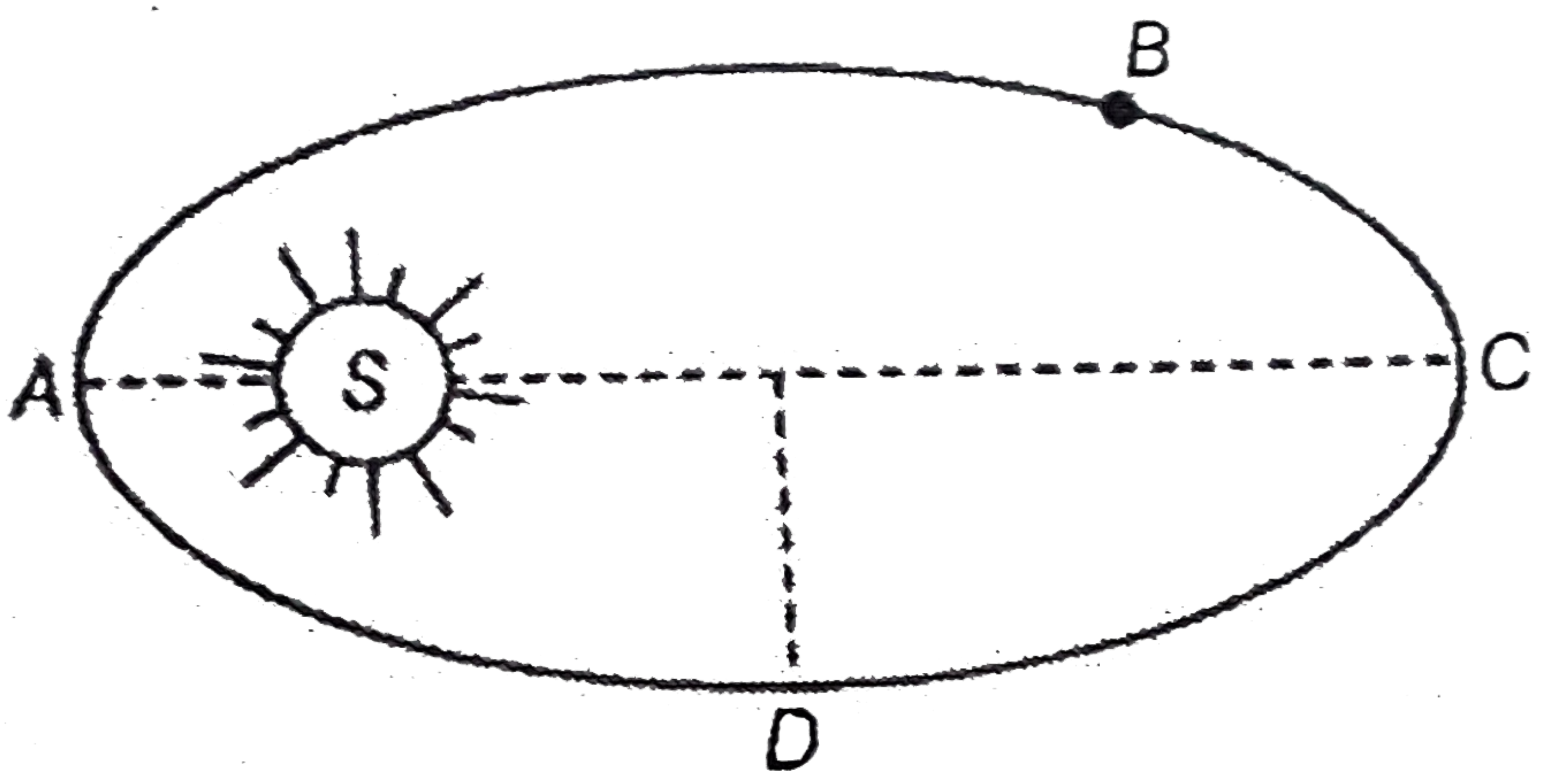
Q-35 - 18254034

A planet revolves in elliptical orbit around the sun. (see figure). The linear speed of the planet will be maximum at



(A) A

(B) B

(C) C

(D) D

CORRECT ANSWER: A

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Q-36 - 17245690

An artificial satellite moving in circular orbit around the earth has total (kinetic + potential) energy E_0. Its potential energy and kinetic energy respectively are :

(A) $2E_0$ and $-2E_0$

(B) $-2E_0$ and $3E_0$

(C) $2E_0$ and $-E_0$

(D) $-2E_0$ and $-E_0$

CORRECT ANSWER: C

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Q-37 - 10964508

A satellite of mass m is just placed over the surface of earth. In this position mechanical energy of satellite is E_1. Now it starts orbiting round the earth in a circular path at height $h =$ radius of earth. In this position, kinetic energy potential energy and total mechancial energy of satellite are K_2, U_2 and E_2 respectively. Then

(A) $U_2 = \frac{E_1}{2}$

(B) $E_2 = \frac{E_1}{4}$

(C) $K_2 = -E_2$

(D) $K_2 = -\frac{U_2}{2}$

CORRECT ANSWER: A::B::C::D

SOLUTION:

$$E_1 = -\frac{GMm}{R}$$

$$U_2 = -\frac{GMm}{2R} \quad (r = R + R = 2R)$$

$$K_2 = \frac{GMm}{4R}$$

and $E_2 = -\frac{GMm}{4R}$

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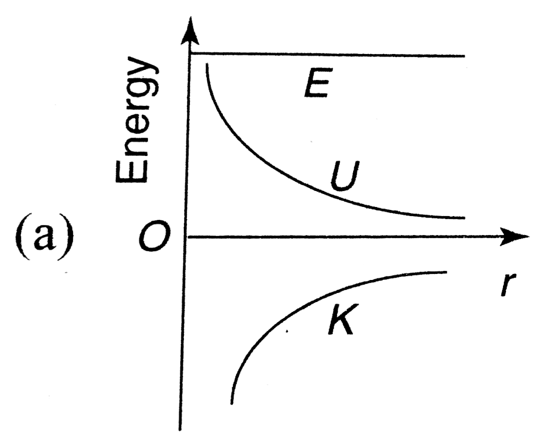
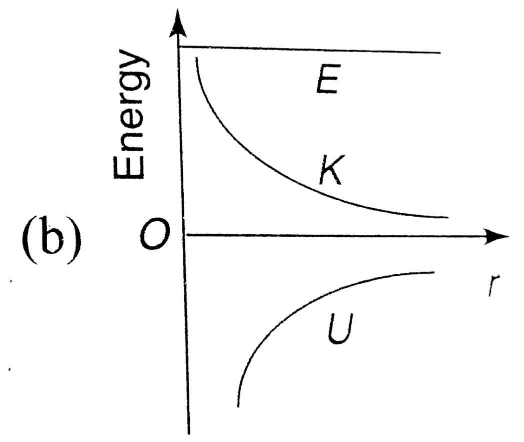
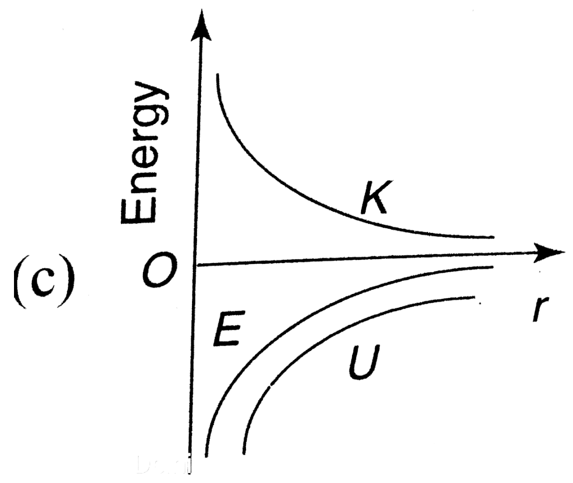
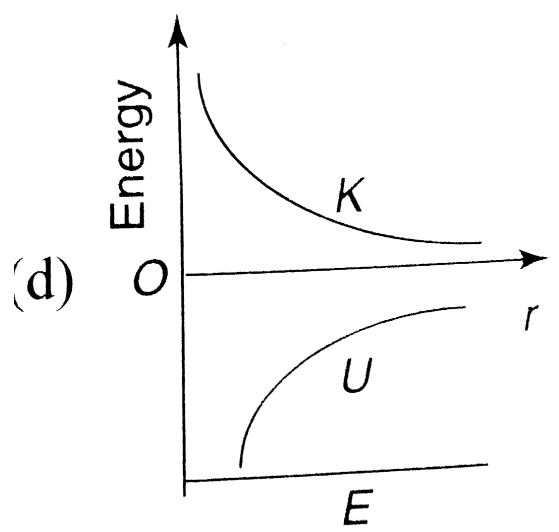
Q-38 - 11748540

The correct graph representing the variation of total energy (E_t), kinetic energy (E_k) and potential energy (U) of a satellite with its distance form the centre of earth is

(A) 

(B) 

(C) 

(D) 

CORRECT ANSWER: C

SOLUTION:

$$U = -\frac{GMm}{r}, K = \frac{GMm}{2r}$$

and $E = \frac{-GMm}{2r}$

For a satellite U, K and E very with r and also U and E remain negative whereas K remain always positive

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Q-39 - 18247454

A solid sphere of mass M and radius R has a spherical cavity of radius R/2 such that the centre of cavity is at a distance R/2 from the centre of the sphere. A point mass m is placed inside the cavity at a distanace R/4 from the centre of sphere. The gravitational force on mass m is

(A) $\dfrac{11GMm}{R^2}$

(B) $\dfrac{14GMm}{R^2}$

(C) $\dfrac{GMm}{2R^2}$

(D) $\dfrac{GMm}{R^2}$

CORRECT ANSWER: C

SOLUTION:

Field strength is uniform inside the cavity. Let us find at its centre

$E_T = E_R + E_C$ (T = Total, R = Remaining, C = Cavity)

$$\therefore E_R = E_T - E_C$$
$$= \frac{GM}{R^3}\frac{R}{2} - 0$$
$$= \frac{GM}{2R^2}$$

$$\therefore F = mF_R$$
$$= \frac{GMm}{2R^2}$$

.

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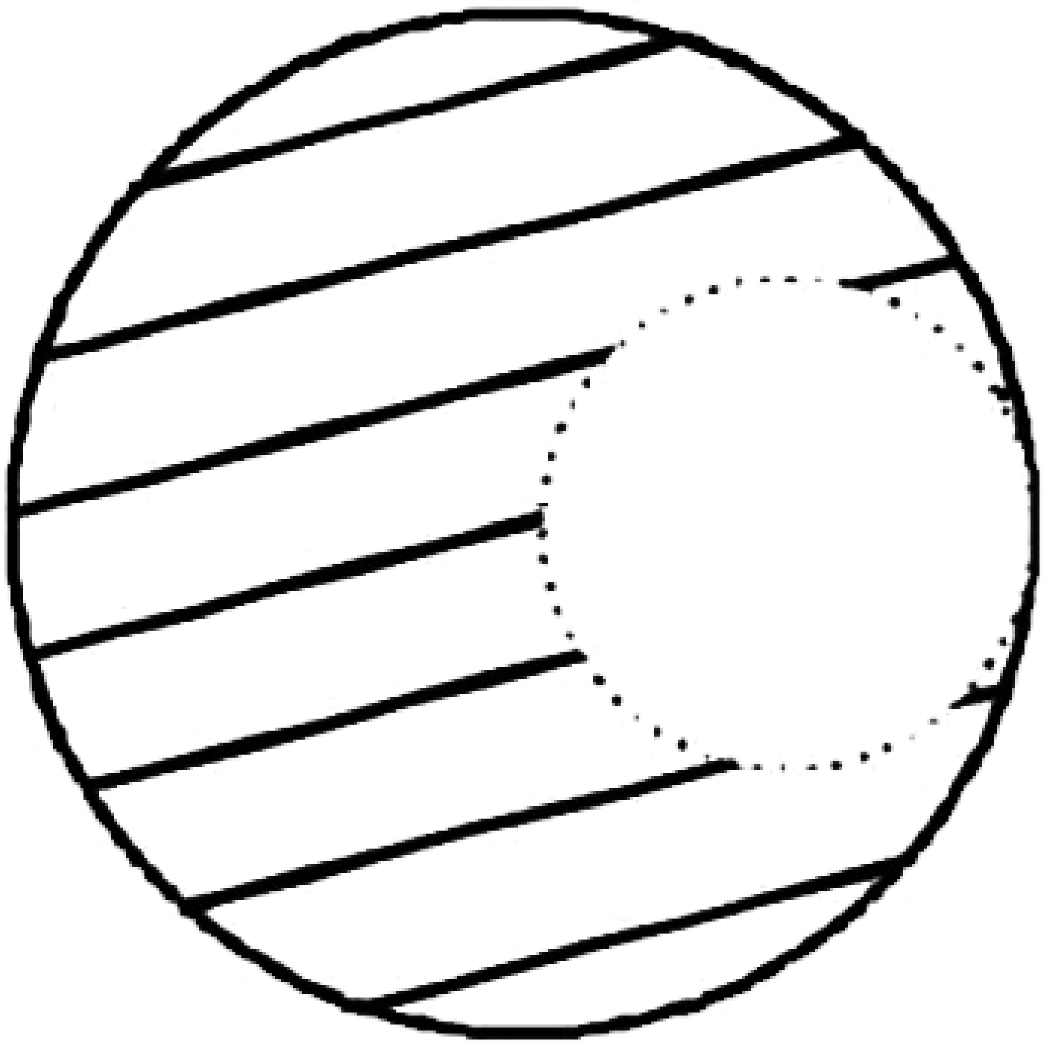
Q-40 - 10058881

From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure Taking gravitational potential $V = 0 at r = \infty,$ the potential at (G = gravitational constatn)



(A) $\frac{-2GM}{3R}$

(B) $\frac{-2GM}{R}$

(C) $\frac{-GM}{2R}$

(D) $\frac{-GM}{R}$

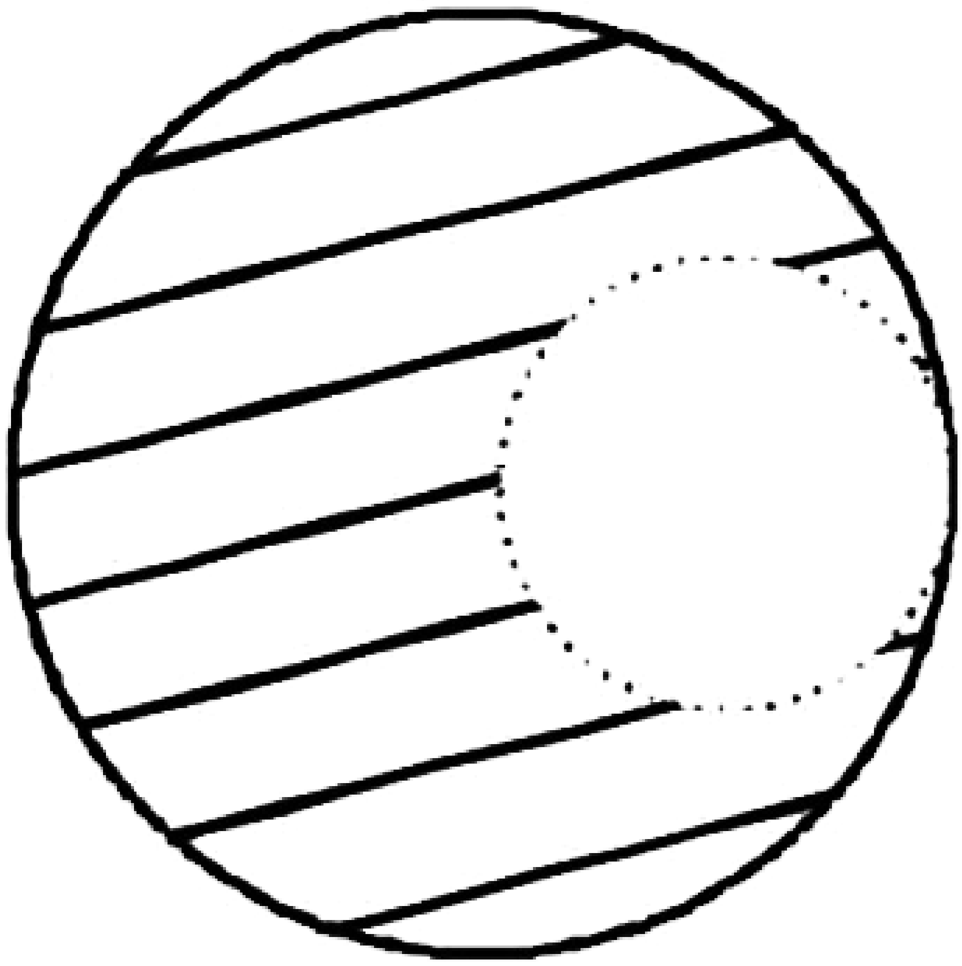
CORRECT ANSWER: D

SOLUTION:

(d) Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[\left(3R^2 - \left(\frac{R}{2}\right)^2\right)\right]$$

$$\frac{-Gm}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{kGM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{8}{R}/(2)$$

$$= -\frac{3GM}{8R}$$

So potential at the centre of cavity

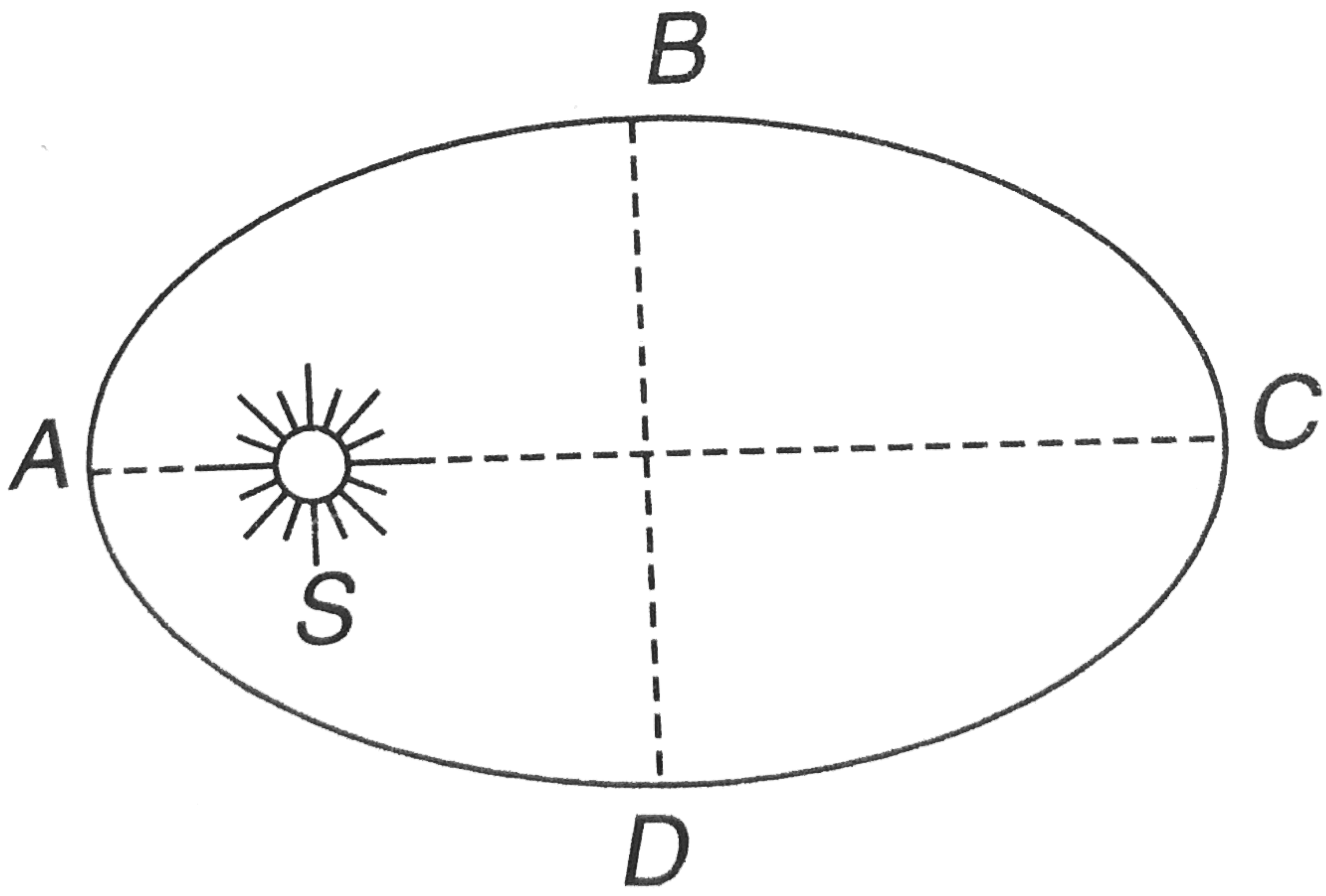
$$= V_{sphere} - V_{cavity} =$$

$$-\frac{11GM}{8R}$$

$$-\left(-\frac{3}{8}\frac{GM}{R}\right)$$

$$= \frac{-GM}{R}$$

A planet is revolving around the sun as shown in elliptical path. The correct option is



(A) The time taken in travelling DAB is less than that for BCD

(B) The time taken in travelling DAB is greater than that for BCD

(C) The time taken in travelling CDA is less than that for

ABC

(D) The time taken in travelling CDA is greater than that for ABC

CORRECT ANSWER: A

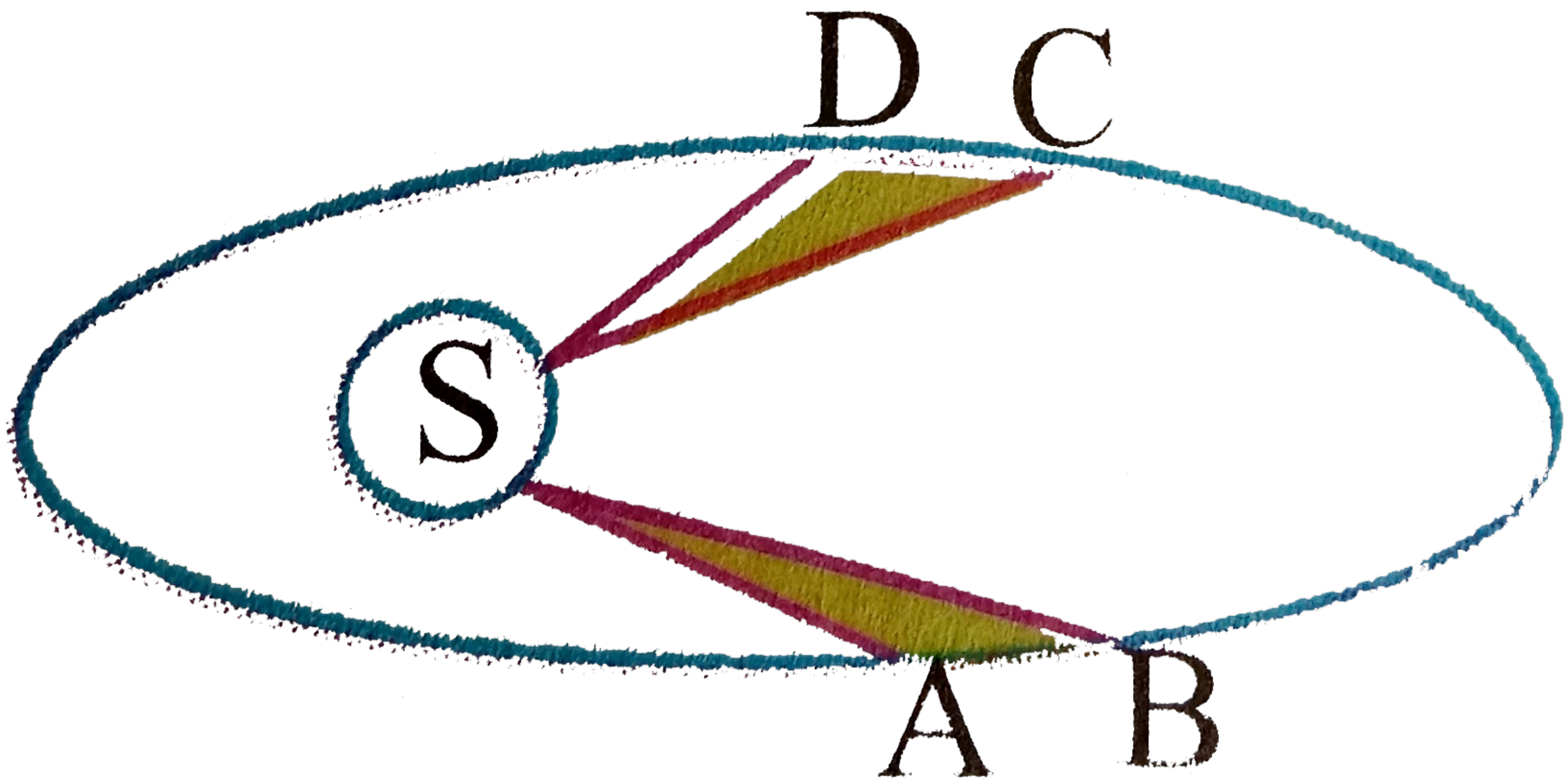
SOLUTION:

During path DAB planet is nearer to sun as comparision with path BCD. So time taken in travelling DAB is less than that for BCD because velocity of planet will be more in region DAB.

Q-42 - 12928254

The motion of a planet around sun in an elliptical orbit is shown in the following figure. Sun is situated at one focus. The shaded areas are equal. If the planet takes time t_1 and l_2 in moving from A to B

and from C to D respectively, then



(A) $t_1 > t_2$

(B) $t_1 < t_2$

(C) $t_1 = t_2$

(D) incomplete information

CORRECT ANSWER: C

Q-43 - 11804511

A planet revolves around the sun in elliptical orbit of semimajor axis 210^{12}m. The areal velocity of the planet when it is nearest to the sun is $4.410^{16} \frac{m^2}{/} s$. The least distance between planet and the sun is 1.810^{12} m. Find the minimum speed of the planet in km/s.

SOLUTION:

Area covered by line joining planet and sun in time dt is

$dS = \frac{1}{2} x^2 dth\eta$ area velocity

$= dS / dt = \frac{1}{2} x^2 dth\eta$

$/ dt = \frac{1}{2} x^2 \omega$

where $x =$ distance between planet and sun

and $\omega =$ angular speed of planet about sum

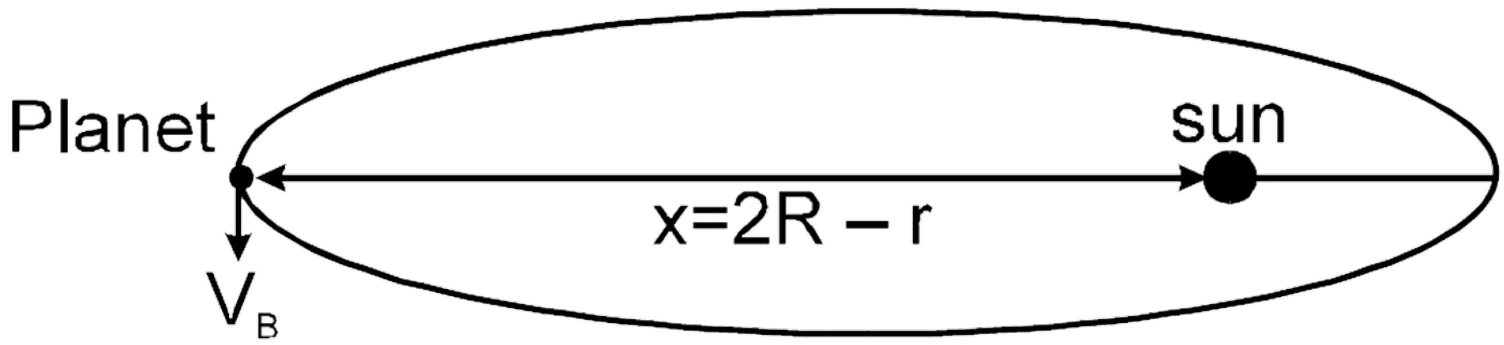
From keplers second law areal velocity of planet is

constant

At farthest position

$$A = dS/dt = \frac{1}{2}(2R$$
$$- r^3)\omega = \frac{1}{2}(2R$$
$$- r)[(2R - r)\omega]$$
$$= \frac{1}{2}(2R - r)V_B$$

$$V_B = 40km/s$$



Q-44 - 12230225

A planet of mass m is the elliptical orbit about the sun $(m < < M_{\text{sun}})$ with an orbital period T. If A be the area of orbit, then its angular momentum would be:

(A) $\frac{2mA}{T}$

(B) mAT

(C) $\frac{mA}{2T}$

(D) $2mAT$

CORRECT ANSWER: A

SOLUTION:

$$\vec{L} = 2m\frac{d\vec{A}}{dt} \Rightarrow L$$

$$= \frac{2dA}{T}$$

Q-45 - 12006811

A planet is revolving around the sun in an elliptical orbit. Which out of the following remains constant.

(a) Linear speed (b) angular momentum

(c) kinetic energy (d) potential energy (e) total energy throughout its orbit.

SOLUTION:

(b) Angular momentum and (e) total energy of the planet, remain constant.

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Q-46 - 14626565

Length of a year on a planet is the duration in which it completes one revolution around the sun. Assume path of the planet known as orbit to be circular with sun at the centre. The length T of a year of a planet orbiting around the sun in circular orbit depends on universal gravitational constant G, mass m_s of the sun and radius r of the orbit. if $T \propto G^a m_s^b r^c$ find value of a+b+2c.

(A) 1

(B) 2

(C) 3

(D) 4

CORRECT ANSWER: B

SOLUTION:

$$[T] = [G]^a [M_5]^b [r]^c$$

MLT

$= [M^{-1}L^3T^{-2}]^a[M$

$]^b[L]^c$

$= M^{b-a}L^{3a+c}T^{-2a}$

$\Rightarrow a = -\frac{1}{2}, b =$

$-\frac{1}{2}, c = \frac{3}{2} \Rightarrow a + b$

$+ 2c = 2$

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Q-47 - 17245726

A hole is drilled from the surface of earth to its centre. A particle is dropped from rest in the surface of earth in terms of its escape velocity on the surface of earth v_e is :

(A) $\frac{v_e}{2}$

(B) v_e

(C) $\sqrt{2}v_e$

(D) $\frac{v_e}{\sqrt{2}}$

CORRECT ANSWER: D

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Q-48 - 15835922

The escape velocity from the surface of earth is V_e. The escape velocity from the surface of a planet whose mass and radius are 3 times those of the earth will be

(A) V_e

(B) $3v_e$

(C) $9V_e$

(D) $27V_e$

CORRECT ANSWER: A

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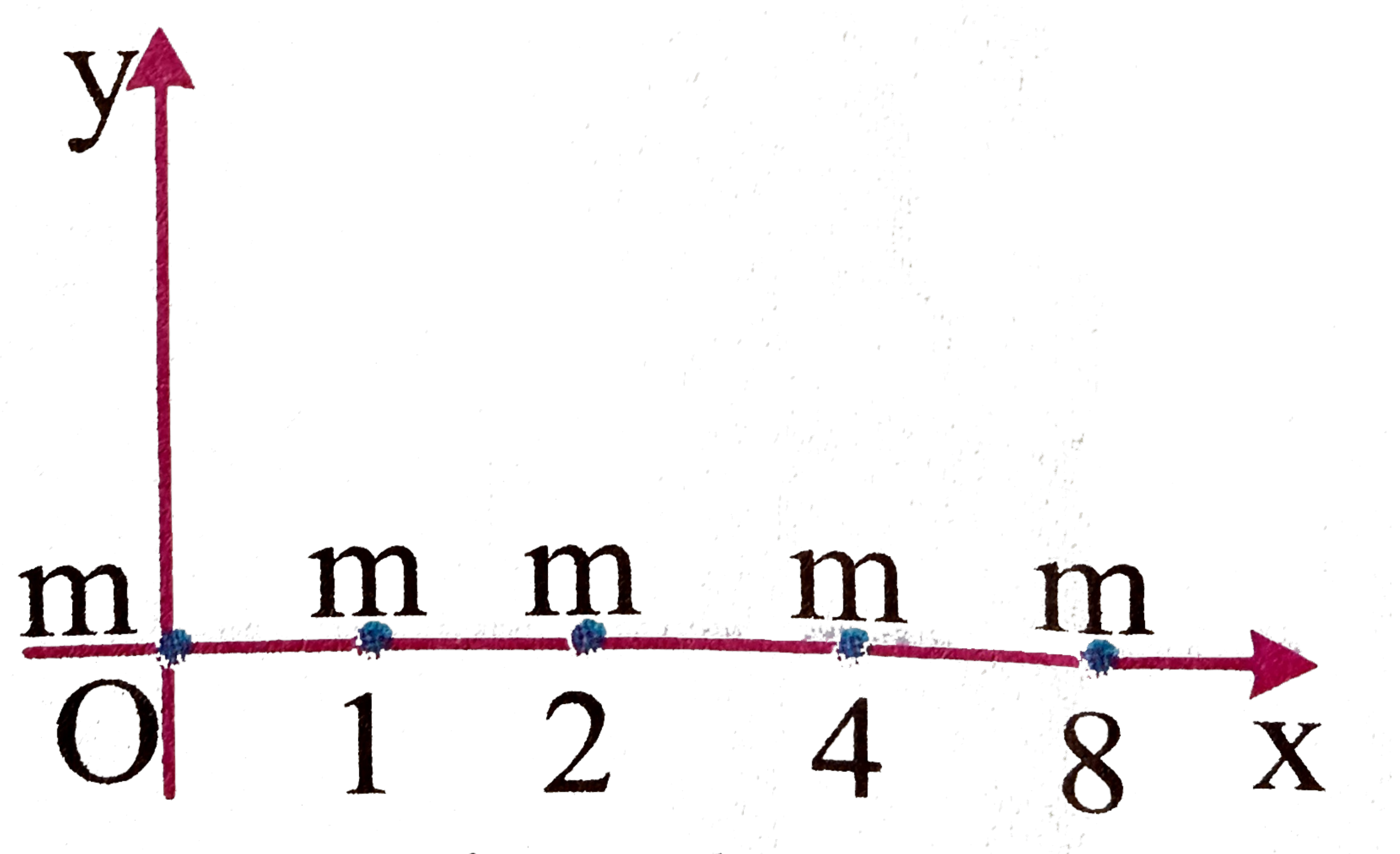
Q-49 - 12928207

An infinite number of particles each of mass m are placed on the positive X-axis of $1m$, $2m$, $4m$, $8m$, ... from the origin. Find the magnitude of the resultant gravitational force on mass m kept at the origin.

SOLUTION:



The resultant gravitational force on m at O is

$$F = \frac{Gm^2}{1} + \frac{Gm^2}{4}$$
$$+ \frac{Gm^2}{16} + \dots.$$

$$= Gm^2\left(1 + \frac{1}{4} + \frac{1}{16}\right.$$
$$\left. + \dots\dots \right)$$

$$= Gm^2\left(\frac{1}{1 - \frac{1}{4}}\right)$$
$$= \frac{4}{3}Gm^2\left(\because S_\infty\right.$$
$$\left. = \frac{a}{1 - r}\right)$$

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Q-50 - 17245648

A particle on earth's surface is given a velocity euqal to its escape velocity. Its total mechanical energy with zero potential energy reference at infinite separation will be:

(A) Negative

(B) Positive

(C) Zero

(D) Infinite

CORRECT ANSWER: C

Q-51 - 14626429

What should be the radius of a planet with mass equal to that of earth and escape velocity on its surface is equal to the velocity of light. Given that mass of earth is $M = 6 \times 10^{24} kg$.

(A) 6mm

(B) 9mm

(C) 18mm

(D) 3mm

CORRECT ANSWER: B

SOLUTION:

$$v_c = \sqrt{\frac{2GM}{R}} \Rightarrow R$$
$$= \frac{2GM}{v_e^2} \approx 9mm$$

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Q-52 - 11302579

A satellite of mass $1000kg$ is rotating around the earth in a circular orbit of radius $3R$. What extra energy should be given to this catellite if it is to be lifted into an orbit of radius $4R$?

SOLUTION:

Energy required $= (TE)_f - (TE)_i$

$$= \left(\frac{-GMm}{2(4R)}\right)$$

$$- \left(\frac{-GMm}{3(3R)}\right)$$

$$= \frac{-Gmm}{8R} + \frac{GMm}{6R}$$

$$= \frac{GMm}{24R}(4$$

$$- 3)\frac{GMm}{24R}$$

$$= \left(\frac{9.8 \times \left(6400 \times 10^3\right)^2 \times 1000}{24\left(6400 \times 110^3\right)}\right)$$

$$= 2.614 \times 10^9 J$$

Q-53 - 14800887

A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius (1.01) R. The period of the second satellite is larger than the first one by approximately

(A) $0.7\ \%$

(B) $1\ \%$

(C) $1.5\ \%$

(D) $3\ \%$

CORRECT ANSWER: C

SOLUTION:

$$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}, T \propto R^{\frac{3}{2}}$$

Radius of 2^{nd} satellite is 1% greater

Hence time period is $1 \times \frac{3}{2} = 1.5\%$ larger

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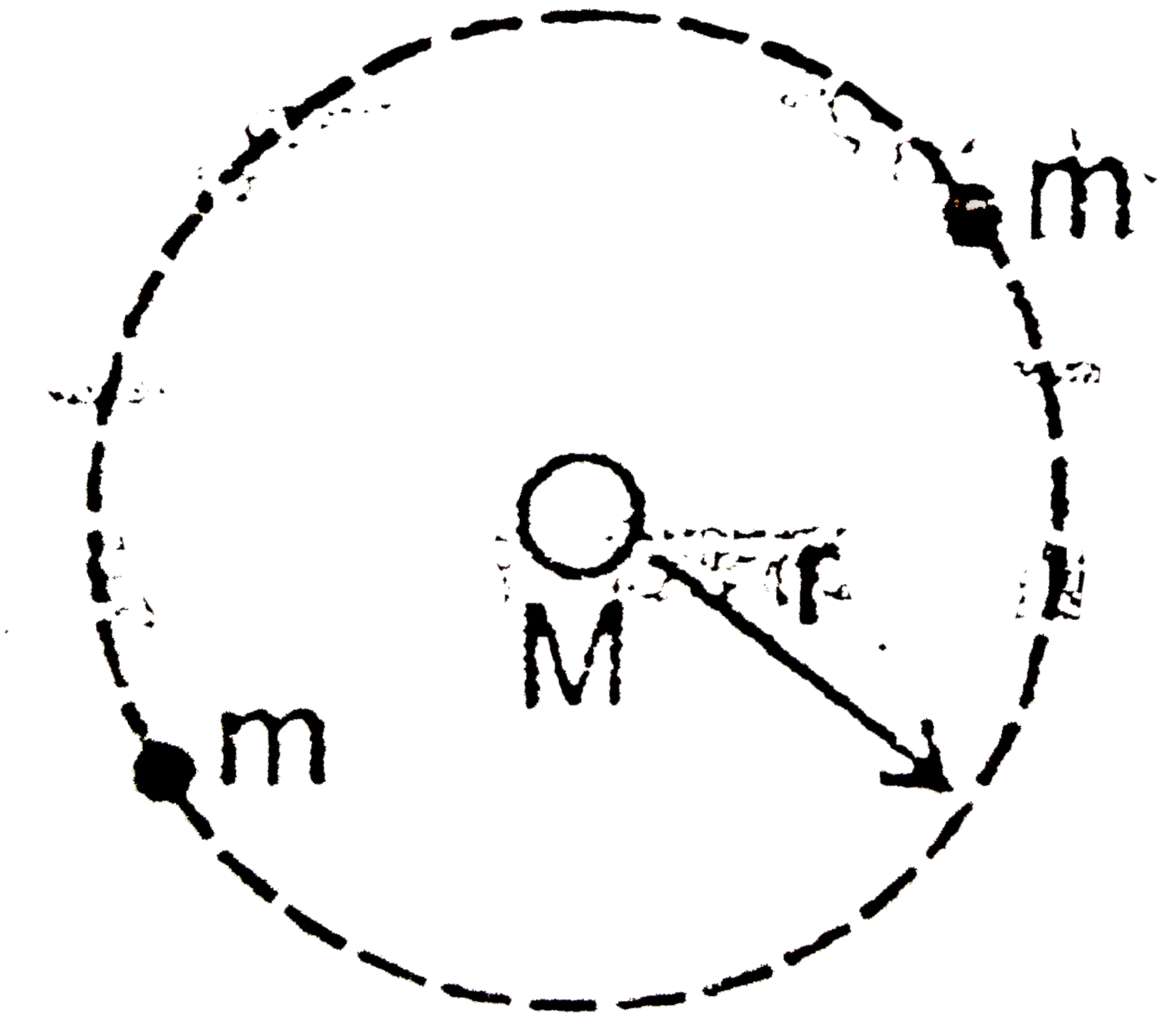
Q-54 - 16113211

An isolated triple star systerm consists of two identical stars, each of mass m and f fixed star of mass M. They revolve around the central star in the same circular orbit to radius r. The orbiting stars are always at opposite ends of a diameter of the orbit. The time period of revolution of each star around the fixed is equal to:



(A) $\frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$

(B) $\frac{2\pi r^{3/2}}{\sqrt{GM}}$

(C) $\frac{2\pi r^{3/2}}{\sqrt{G(M+m)}}$

(D) $\frac{4\pi r^{3/2}}{\sqrt{G(M+m)}}$

CORRECT ANSWER: A

SOLUTION:

Net gravitational force on any orbitin star provides

necessary centripetal force.

So,

$$\frac{GMm}{r^2} + \frac{Gm^2}{4r^2}$$

$$= \frac{mV^2}{r}$$

$$\Rightarrow V$$

$$= \sqrt{\frac{G(4M+m)}{4r}}$$

$$T = \frac{2\pi r}{V}$$

$$= \frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$$

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Q-55 - 12928363

If V, T, L, K and r denote speed, time period, angular momentum, kinetic energy and radius of satellite in circular orbit (a)$V \alpha r^{-1}$,(b) $L \alpha r^{1/2}$

(c)$T \alpha r^{3/2}$,(d) $K \alpha r^{-2}$

(A) a, b are true

(B) b, c are true

(C) a, b, d are true

(D) a, b, c are true

CORRECT ANSWER: B

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Q-56 - 10964358

Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. if the gravitational force of attraction between the planet and the star is proportational to $R^{-5/2}$, then

(a) T^2 is proportional to R^2

(b) T^2 is proportional to $R^{7/2}$

(c) T^2 is proportional to $R^{3/3}$

(d) T^2 is proportional to $R^{3.75}$.

CORRECT ANSWER: A::B::C::D

SOLUTION:

$$\frac{mv^2}{R} \propto R^{-5/2}$$

$$\therefore v \propto R^{-3/4}$$

Now, $T = \dfrac{2\pi R}{v}$ or $T^2 \propto \left(\dfrac{R}{v}\right)^2$

or $T^2 \propto \left(\dfrac{R}{R^{-3/4}}\right)^2$ or $T^2 \propto R^{\frac{7}{2}}$

The correct answer is (b).

Q-57 - 12928638

The angular momentum (L) of the earth revolving round the sun uis proportional to r^n, where r is the orbital radius of the earth. The value of n is (assume the orbit to be circular)

(A) $\frac{1}{2}$

(B) 1

(C) $-\frac{1}{2}$

(D) 2

CORRECT ANSWER: A

SOLUTION:

$$L = mvr, \frac{mv^2}{r}$$

$$= \frac{GMm}{r^2}$$

Q-58 - 15836219

Time period of revolution of a nearest satellite around a planet of radius R is T . Period of revolution around another planet, whose radius is 3R but having same density is

(A) T

(B) $3T$

(C) $9T$

(D) $3\sqrt{3}T$

CORRECT ANSWER: A

Q-59 - 12230207

A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is $1/2$ of g at poles. The escape velocity of a particle on the planet in terms of V.

(A) $V_e = 2V$

(B) $V_e = V$

(C) $V_e = V/2$

(D) $V_e = \sqrt{3}V$

CORRECT ANSWER: A

SOLUTION:

$$g_e = g_p - R\omega^2 \Rightarrow \frac{g}{2}$$

$$= g - R\omega^2$$

$$R\omega^2 = \frac{g}{2} \Rightarrow R^2\omega^2 = \frac{gR}{2}$$

$$V^2 = \frac{gR}{2} \ldots . (1)$$

$$V_e = \sqrt{2gR} \ldots . (2)$$

From (1) and (2)

$$V_e = \sqrt{2 \times 2V^2} \Rightarrow V_e = 2V$$

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Q-60 - 22674321

If G is the uciversal gravitational constant and p is the uniform density of a spherical planet. Then shortest possible period o0f rotation around a planet can be

(A) $\sqrt{\frac{\pi G}{2p}}$

(B) $\sqrt{\frac{3\pi GP}{p}}$

(C) $\sqrt{\frac{\pi}{6Gp}}$

(D) $\sqrt{\frac{3\pi}{Gp}}$

CORRECT ANSWER: D