Q-1 - 11748414

Consider a rod of mass M and length L pivoted at its centre is free to rotate in a vertical position plane. The rod is at rest in the vertical position. A bullet of mass M moving horizontally at a speed v strikes and embedded in one end of the rod. The angular velocity of the rod just after the collision will be.

(A) v / L

(B) $2v / L$

(C) $3v / 2L$

(D) $6v / L$

CORRECT ANSWER: C

SOLUTION:

(c) Apply conservation of angular momentum about C

$$mv\frac{L}{2}=I\omega, \text{ where } I=M\left(\frac{L}{2}\right)^2+\frac{ML^2}{12}$$

$$\Rightarrow \omega=\frac{3v}{2L}.$$

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Q-2 - 16979247

A uniform rod of mass m, hinged at its upper end, is released from rest from a horizontal position. When it passes through the vertical position, the force on the hinge is

(A) $\frac{3}{2}mg$

(B) $2mg$

(C) $\frac{5}{2}mg$

(D) 3mg

CORRECT ANSWER: C

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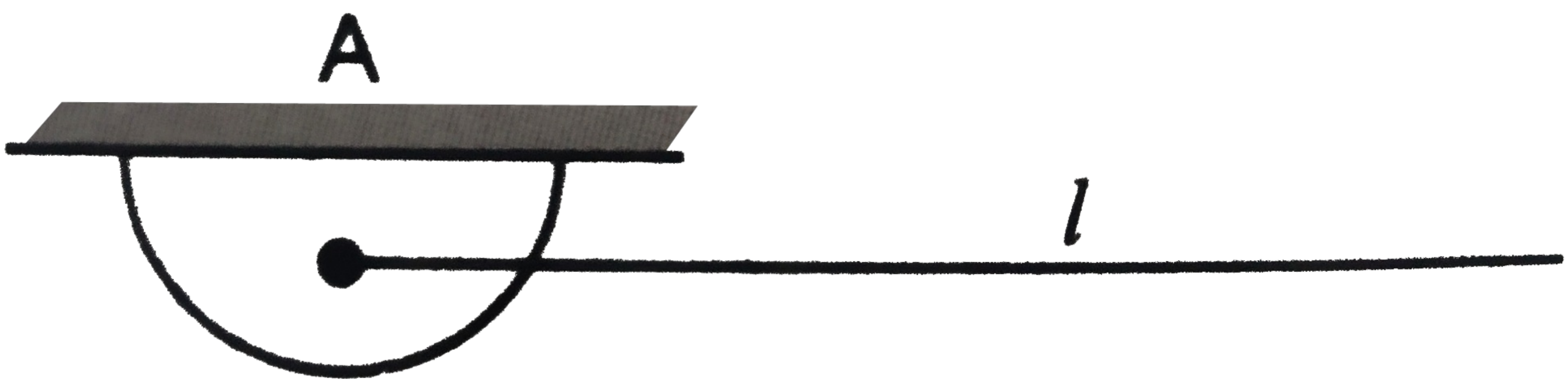
Q-3 - 11765347

A unifrom rod of length l and mass m is free to rotate in a vertical plane about A as shown in Fig. The rod initially in horizontal position is released. The initial angular acceleration of the rod is



(A) $\frac{3g}{2l}$

(B) $\frac{3g}{l}$

(C) $\frac{g}{2l}$

(D) $\frac{2g}{l}$

CORRECT ANSWER: A

SOLUTION:

From $I\alpha = \tau = F \times r$

$$\frac{ml^2}{3}\alpha = mg \times \frac{l}{2}$$

$$\alpha = \frac{3g}{2l}$$

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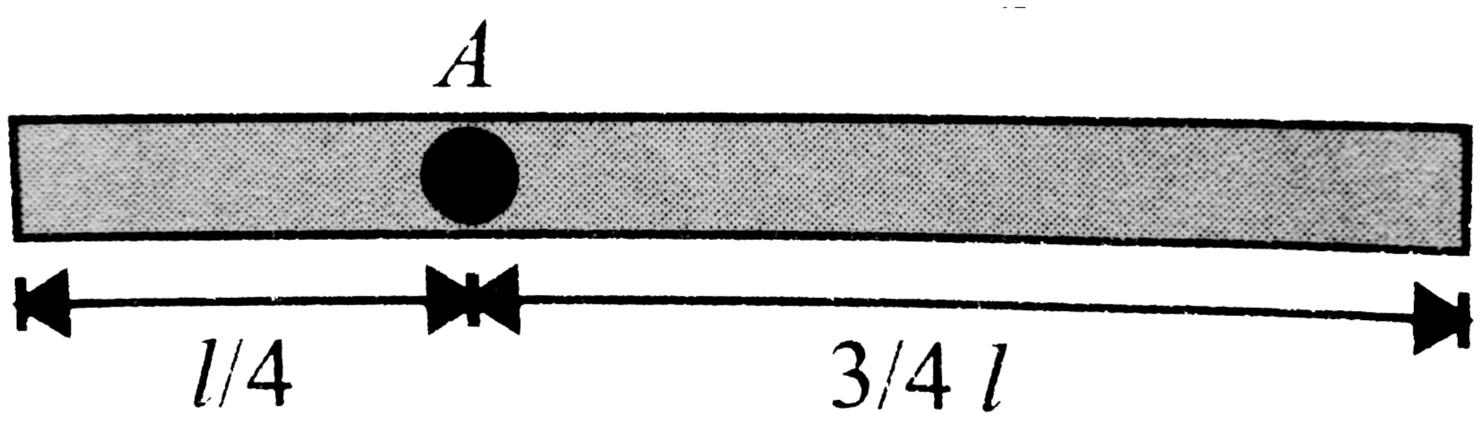
Q-4 - 11301208

A uniform rod of mass m and length l is fixed from Point A, which is at a distance $l/4$ from one end as shown in the figure. The rod is free to rotate in a vertical plane. The rod is released from the horizontal position.



What is the reaction at the hinge, when kinetic energy of the rod is maximum?

(A) $\frac{4}{7}$

(B) $\frac{5}{7}mg$

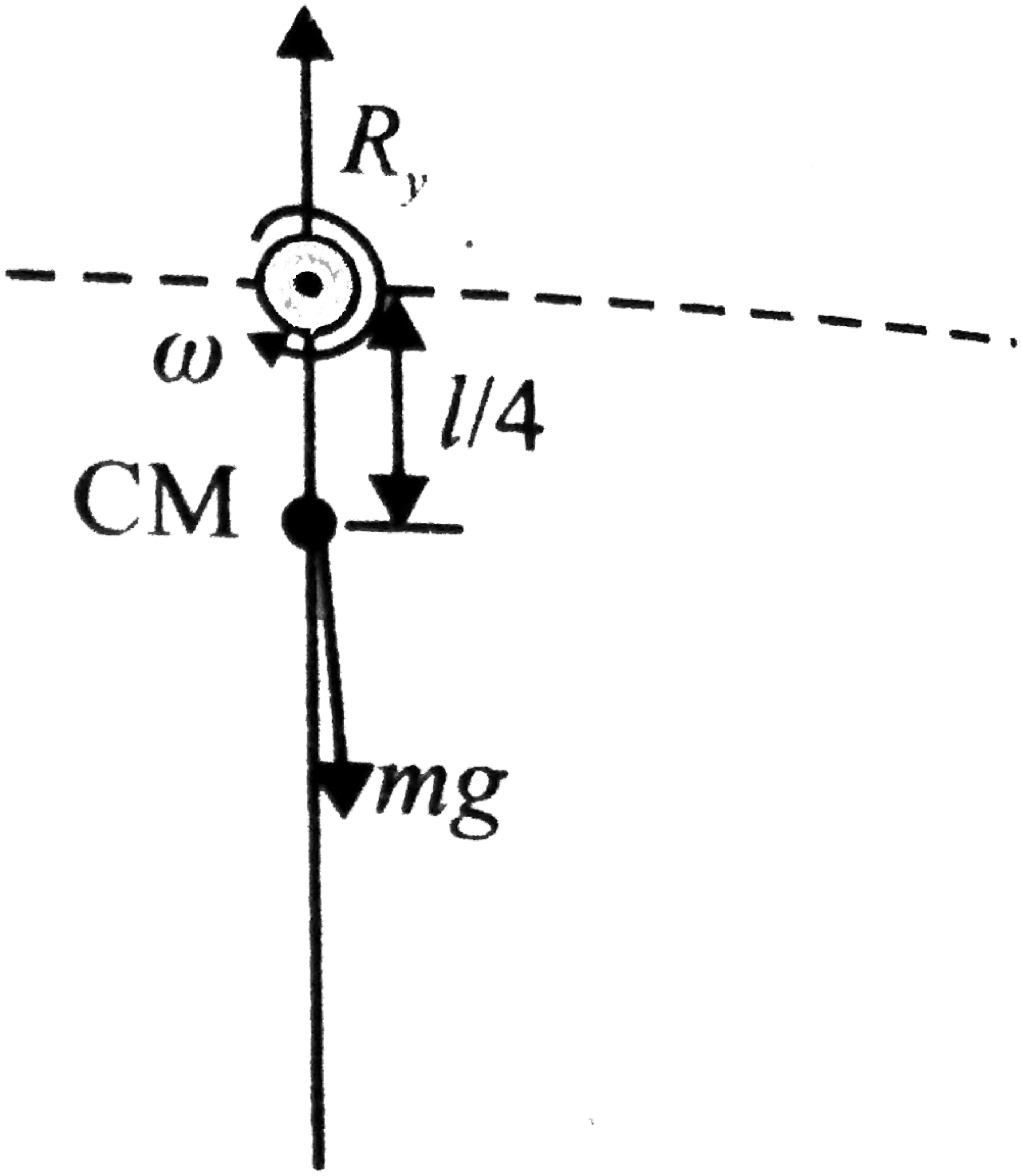
(C) $\frac{13}{7}mg$

(D) $\frac{11}{7}mg$

CORRECT ANSWER: C

SOLUTION:

KE of the rod is maximum when it is in the vertical position. From coservation of energy we get



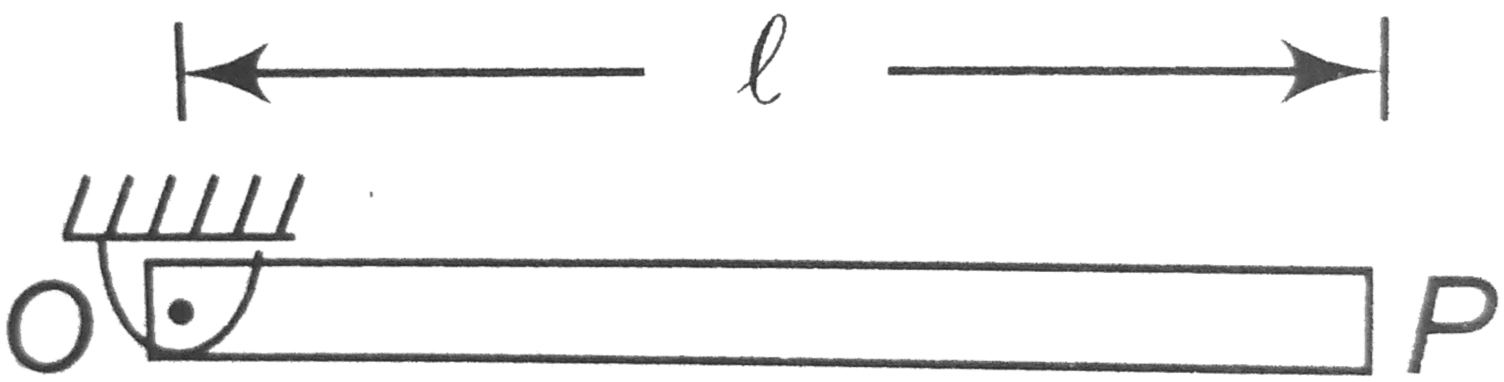
$$\frac{mgl}{4} = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{16}\right]\omega^2$$

$$\frac{\omega^2 l}{4} = \frac{6g}{7}$$

$$R_y = mg + m\left(\frac{6g}{7}\right) = \frac{13mg}{7}$$

Q-5 - 11748175

A uniform rod smoothly pivoted at one of its ends is released from rest. If it swings in vertical plane, the maximum speed of the end P of the rod is.



(A) $2\sqrt{3gl}$

(B) $\sqrt{3gl}$

(C) $2\sqrt{2gl}$

(D) $\sqrt{gl}$

CORRECT ANSWER: B

SOLUTION:

(b) At the time of maximum speed of point P. The rod should be vertical

$$\Delta K + \Delta U = 0$$

$$\left[\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 - 0\right] + \left[-mg\left(\frac{l}{2}\right)\right] = 0$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

Velocity of point $P V_P = \omega l = \sqrt{3gl}$.

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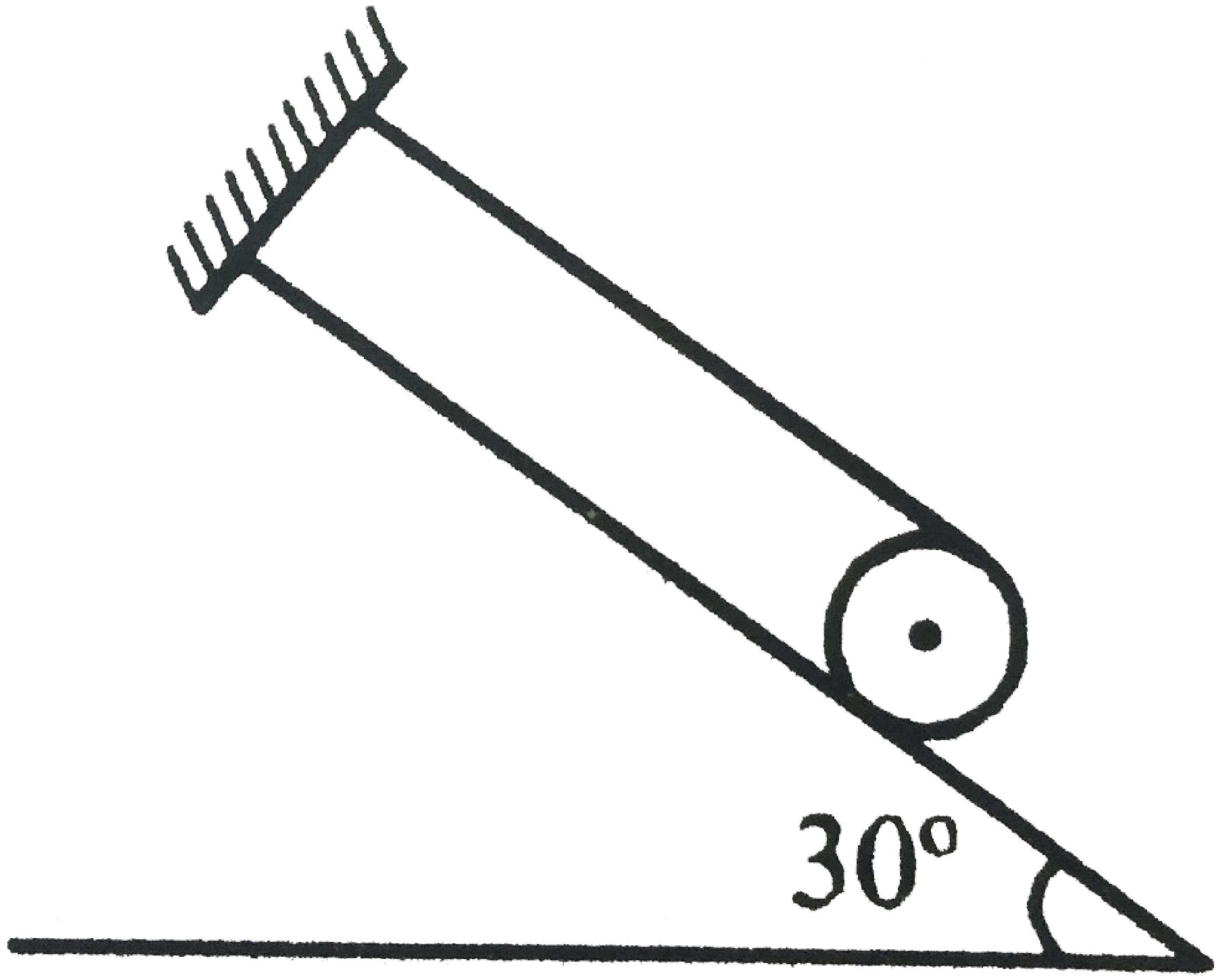
Q-6 - 12229760

A thin hoop of weight $500N$ and radius $1m$ rest on a rought inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration is.



(A) $\frac{1}{3\sqrt{3}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$

(D) $\frac{1}{2\sqrt{3}}$

CORRECT ANSWER: D

SOLUTION:

Taking translational equilibrium along plane.

$T + F_l = mg\sin 30$.(1)

Taking translating equilibrium $\perp$ to the plane

$N = mg\cos 30$

$F_l = \mu N \Rightarrow F_l$

$= \mu mg\cos 30$

(2)

For rotational equilibrium about centre of mass

$T \times R = F_l \times R, T$

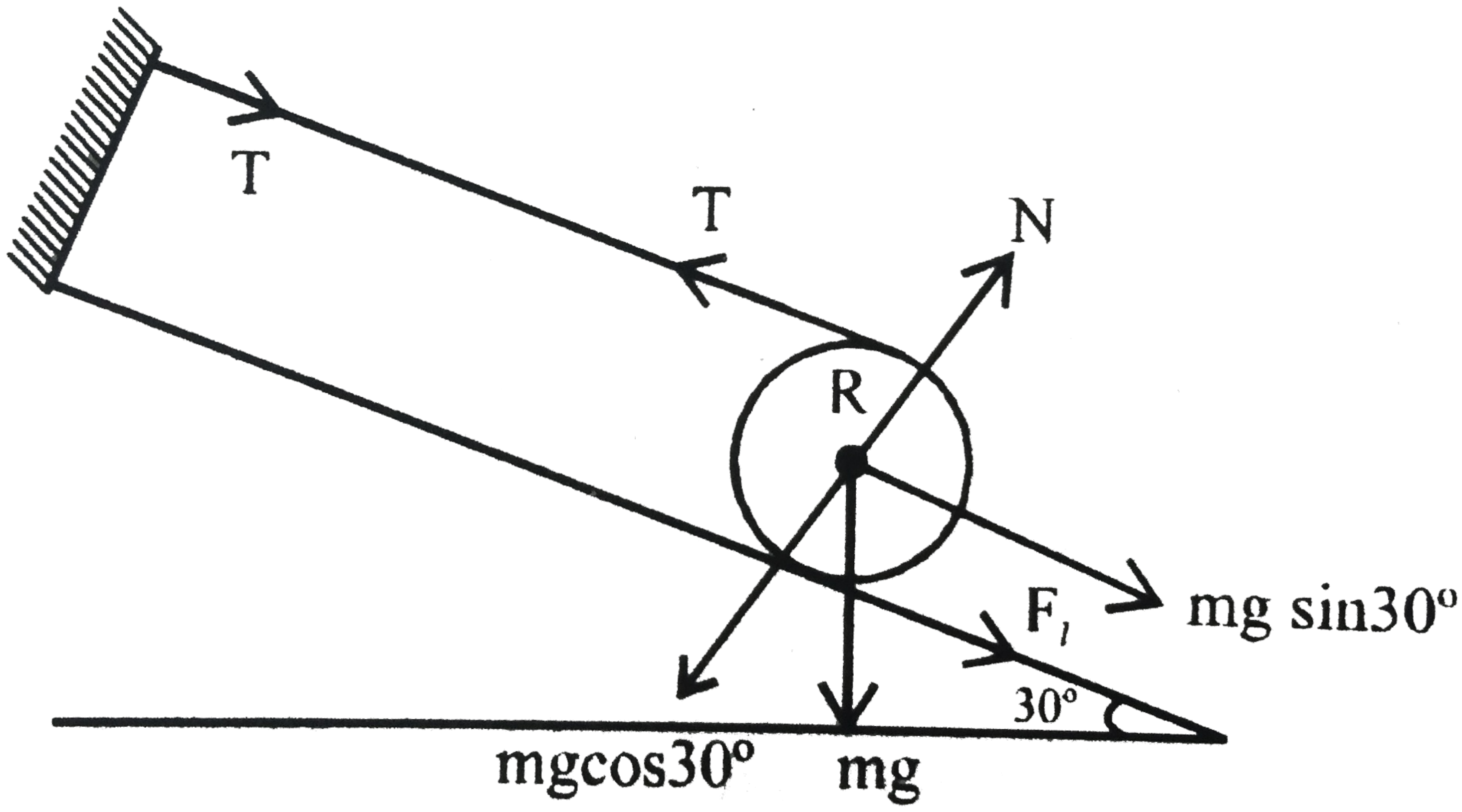
$= F_l$

....(3)

From (1) and (3)

$2F_l = mg\sin 30$ ....(4)

From (2) and (4)



2

$\times\ \mu mg\cos 30^{=} mg\sin 30$

$\mu = \frac{\tan 30^{\square}}{2} \Rightarrow \mu$

$= \frac{1}{2\sqrt{3}}$

.

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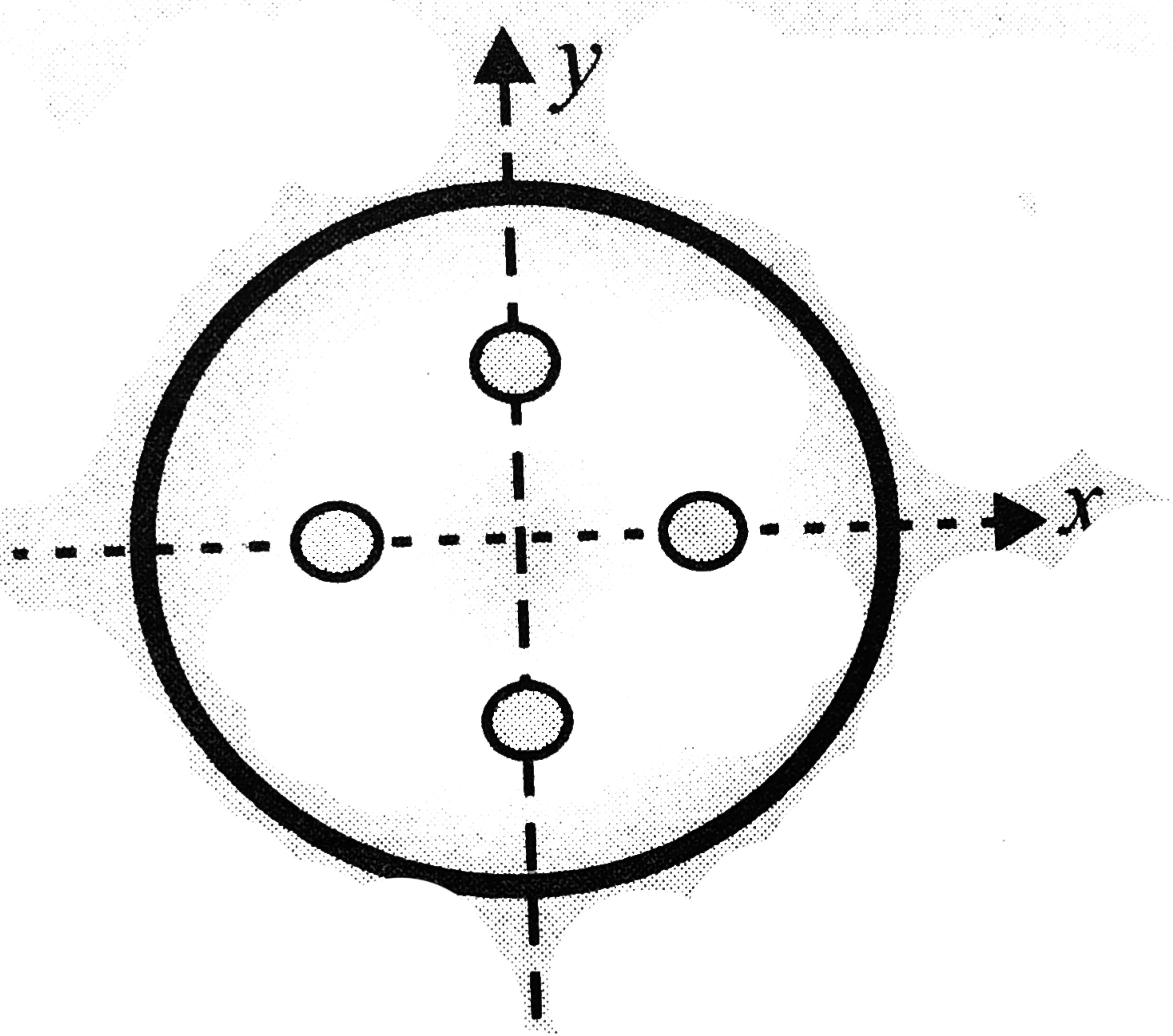
Q-7 - 11301109

A uniform disc of mass m and radius R has an additional rim of

mass m as well as four symmetrically placed masses, each of mass

$m/4$ tied at positions $R/2$ from the centre as shown in Fig. What is the total moment of inertia of the disc about an axis perpendicular to the disc through its centre?



SOLUTION:

$$I = I_{\text{disc}} + I_{\text{ring}} + I_{\text{point masses}}$$

$$= \frac{mR^2}{2} + mR^2$$

$$+ \left[\frac{m}{4} + \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{7mR^2}{4}$$

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Q-8 - 17458907

Two men each of mass m stand on the rim of a horizontal circular disc, diametrically opposite to each other. The disc has a mass M and is free to rotate about a vertical axis passing through its centre of mass. Each mass start simultaneously along the rim clockwise and reaches their original starting positions on the disc. The angle turned through by disc with respect to the ground (in radian) is

(A) $\frac{8m\pi}{4m + M}$

(B) $\frac{2m\pi}{4m + M}$

(C) $\frac{m\pi}{M+m}$

(D) $\frac{4m\pi}{2M+m}$

CORRECT ANSWER: A

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Q-9 - 15599906

The moment of inertia of a disc of mass M and radius R about a tangent to its rim in its plane is

(A) $\frac{2}{3}MR^2$

(B) $\frac{3}{2}MR^2$

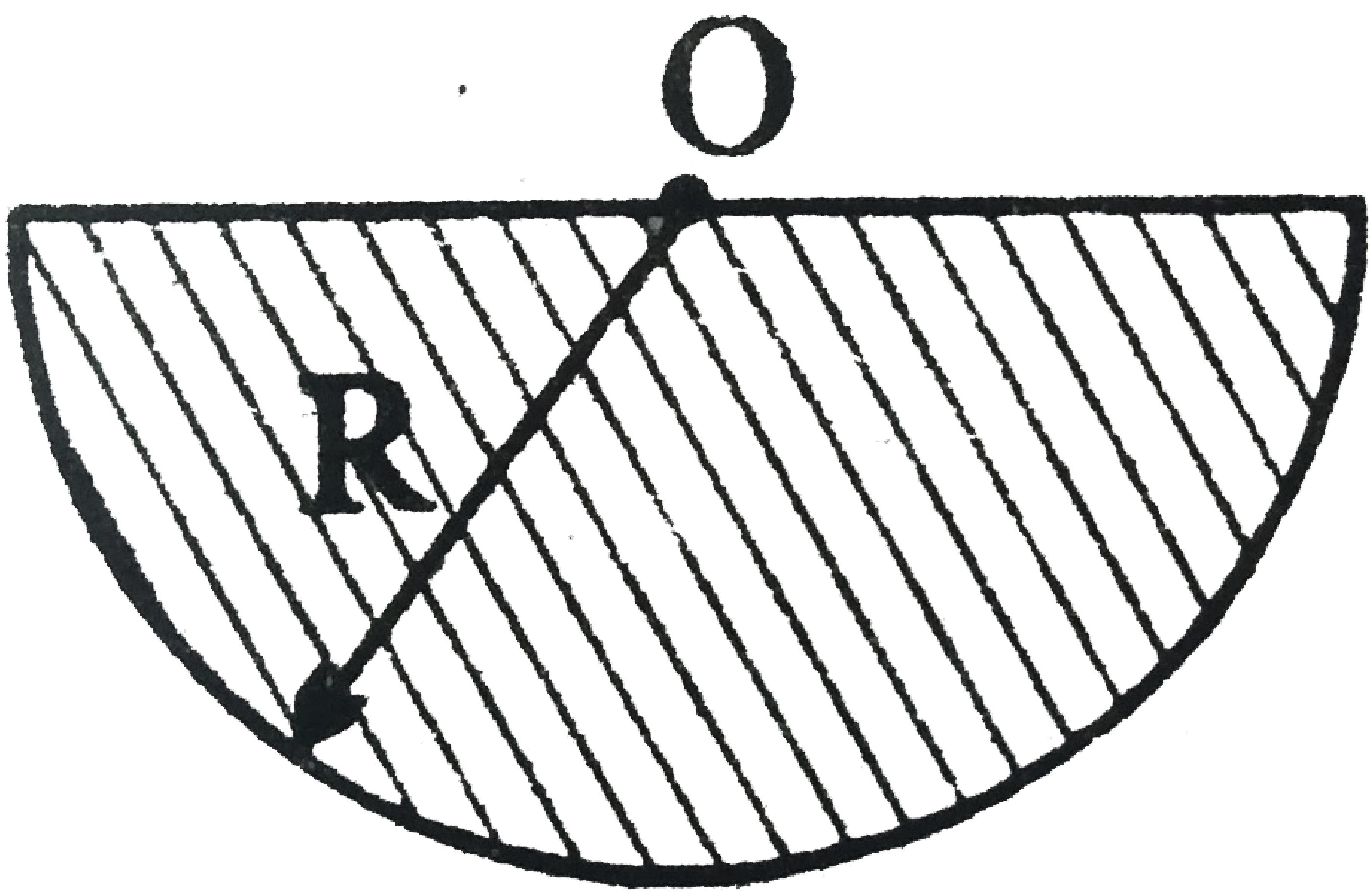
(C) $\frac{4}{5}MR^2$

(D) $\frac{5}{4}MR^2$

CORRECT ANSWER: D

Q-10 - 12229749

Moment of inertia of a thin semicircular disc $(mass - M \& radius = R)$ about an axis through point O and perpendicular to plane of disc, is given by :



(A) $\frac{1}{4}MR^2$

(B) $\frac{1}{2}MR^2$

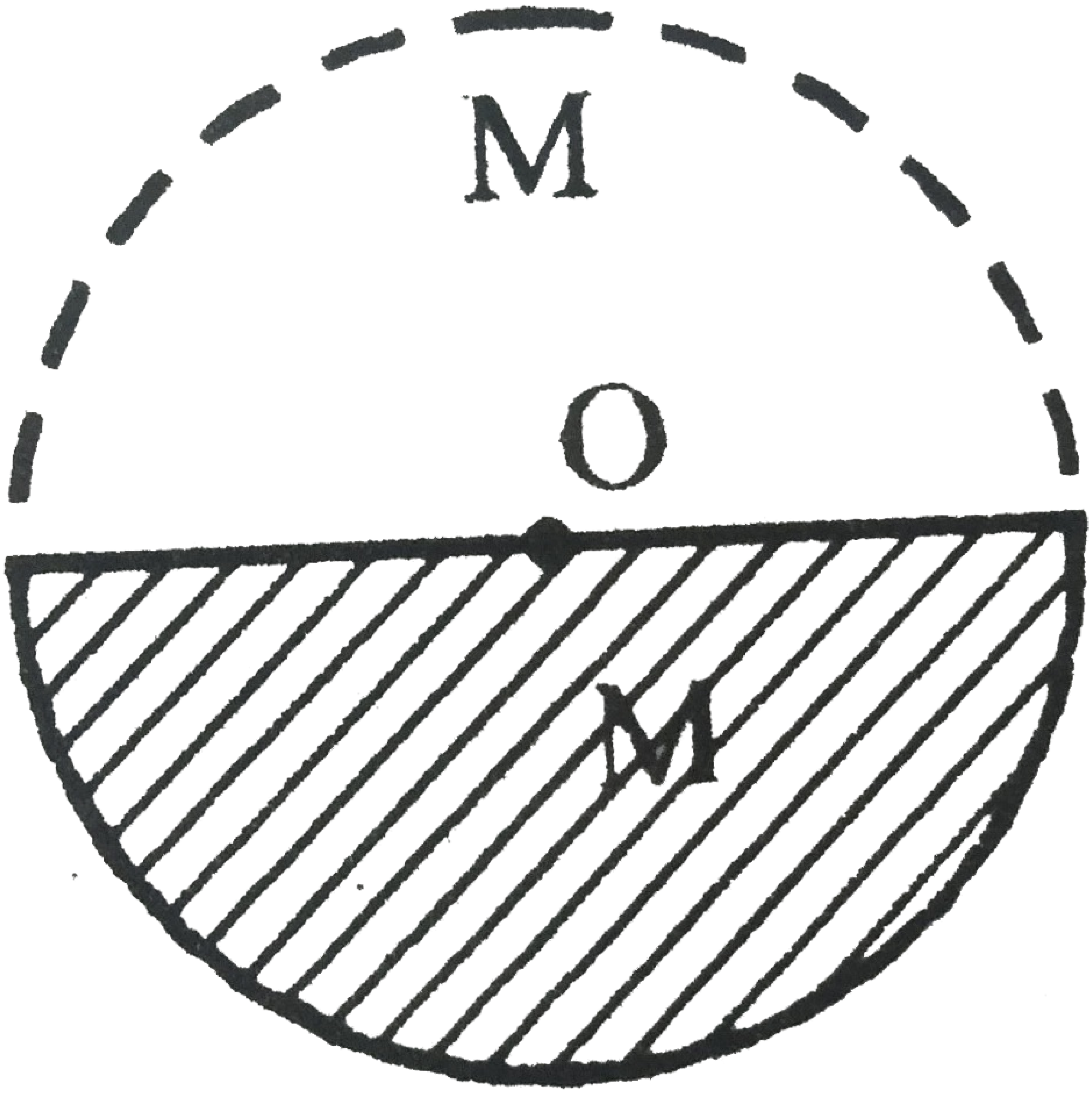
(C) $\frac{1}{8}MR^2$

(D) MR^2

CORRECT ANSWER: B

SOLUTION:

$$I_{disc} = \frac{1}{2}(2M)R^2$$
$$I_{Halfdisc} = \frac{1}{2}MR^2.$$



Q-11 - 17458924

A uniform stick of length l and mass m lies on a smooth table. It rotates with angular velocity ω about an axis perpendicular to the table and through one end of the stick. The angular momentum of the stick about the end is

(A) $ml^2\omega$

(B) $\dfrac{ml^2\omega}{3}$

(C) $\dfrac{ml^2\omega}{12}$

(D) $\dfrac{ml^2\omega}{6}$

CORRECT ANSWER: B

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Q-12 - 10964033

A rod of mass 2 kg ad length 2 m is rotating about its one end O

wth an angular velocity $\omega = 4rad/s$. Find angular momentum of the rod about the axis rotation.

SOLUTION:

In pure rotational motion of a rigid body, component of total angualr momentum about axis of rotation is given by

$$L = I\omega = \left(\frac{ml^2}{3}\right)\omega \left(I_0 = \frac{ml^3}{3}\right)$$

Substituting the value we have,

$$L = \frac{(2)(2)^2}{3}(4)$$
$$= \frac{32}{3}kg - m^2/s$$

direction of this component is perperdicular to paper inwards (from right hand rule), as the rotation clockwise.

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Q-13 - 18707741

A rigid and a disc of different masses are rotating with the same kinetic energy. If we apply a retarding torque τ on the ring, it stops after making n revolution. After how many revolutions will the disc stop, if the retarding torque on it is also τ ?

(A) $\frac{n}{2}$

(B) n

(C) $2n$

(D) Data insufficient

CORRECT ANSWER: B

SOLUTION:

Work done by retarding torque = change in kinetic energy Since, kinetic energy of both are equal, both will rotate with same revolution.

Q-14 - 17091770

A disc with moment of inertiaI_1, is rotating with an angular velocity ω_1 about a fixed axis. Another coaxial disc with moment of inertia I_2, which is at rest is gently placed on the first disc along the axis. Due to friction discs slips against each other and after some time both disc start rotating with a common angular velocity. Find this common angular velocity.

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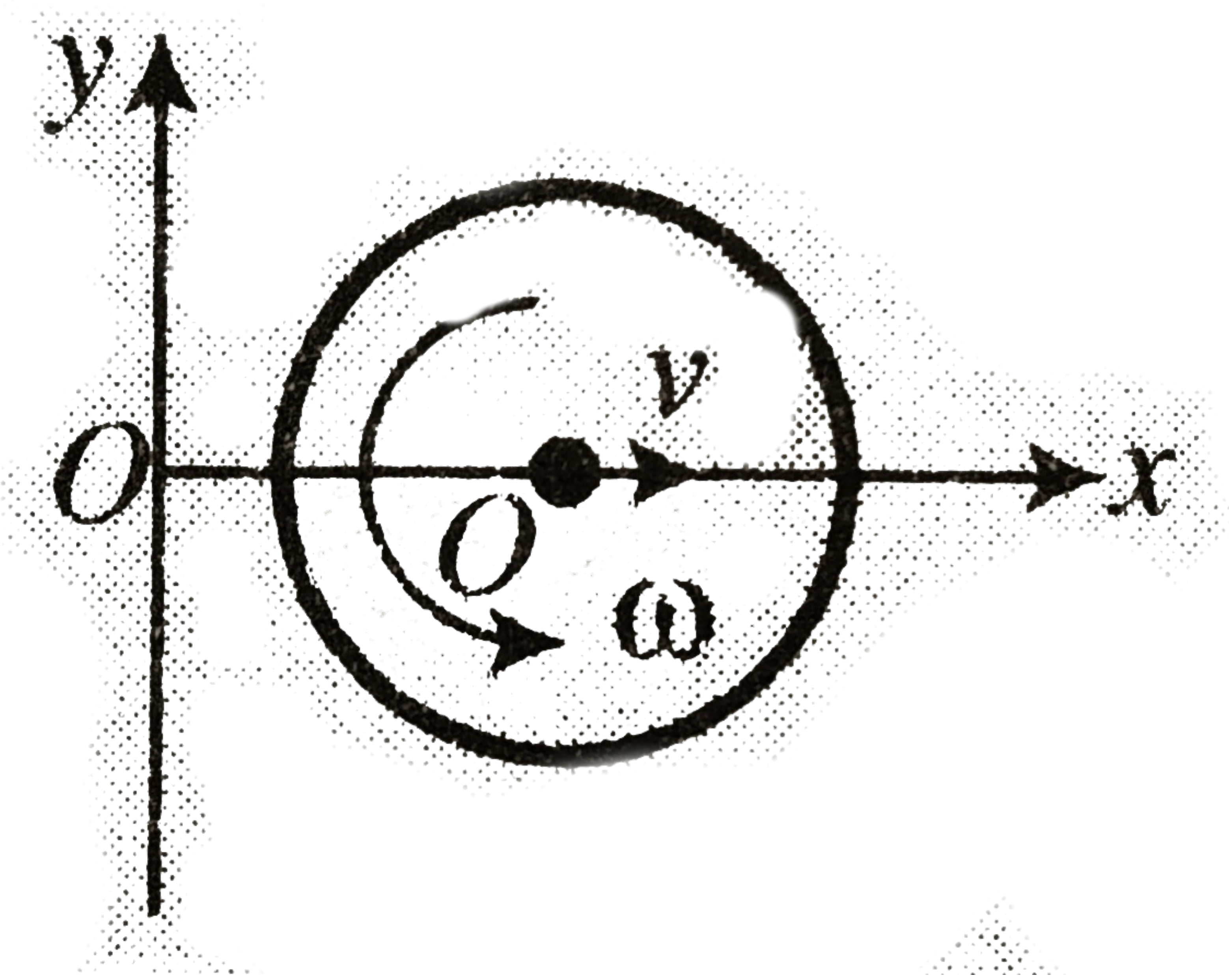
Q-15 - 11301092

A rotating disc moves in the positive direction of x-axis as shown. Find the equation $y(x)$ describing the position of the instantaneous axis of rotation if at the initial moment the centre C of the disc was located at origin after which

a. it moved with constant acceleration a (initial velocity zero) while the disc rotating anticlockwise with constant angular velocity ω. b. it moved with constant velocity v while the disc started rotating anticlockwise with a constant angular acceleration a (with initial angular velocity zero).



SOLUTION:

Let the coordinate of instantaneous centre of rotation at any time t be (x,y).

Now, $v = \omega y$

$\sqrt{2ax} = \omega y$

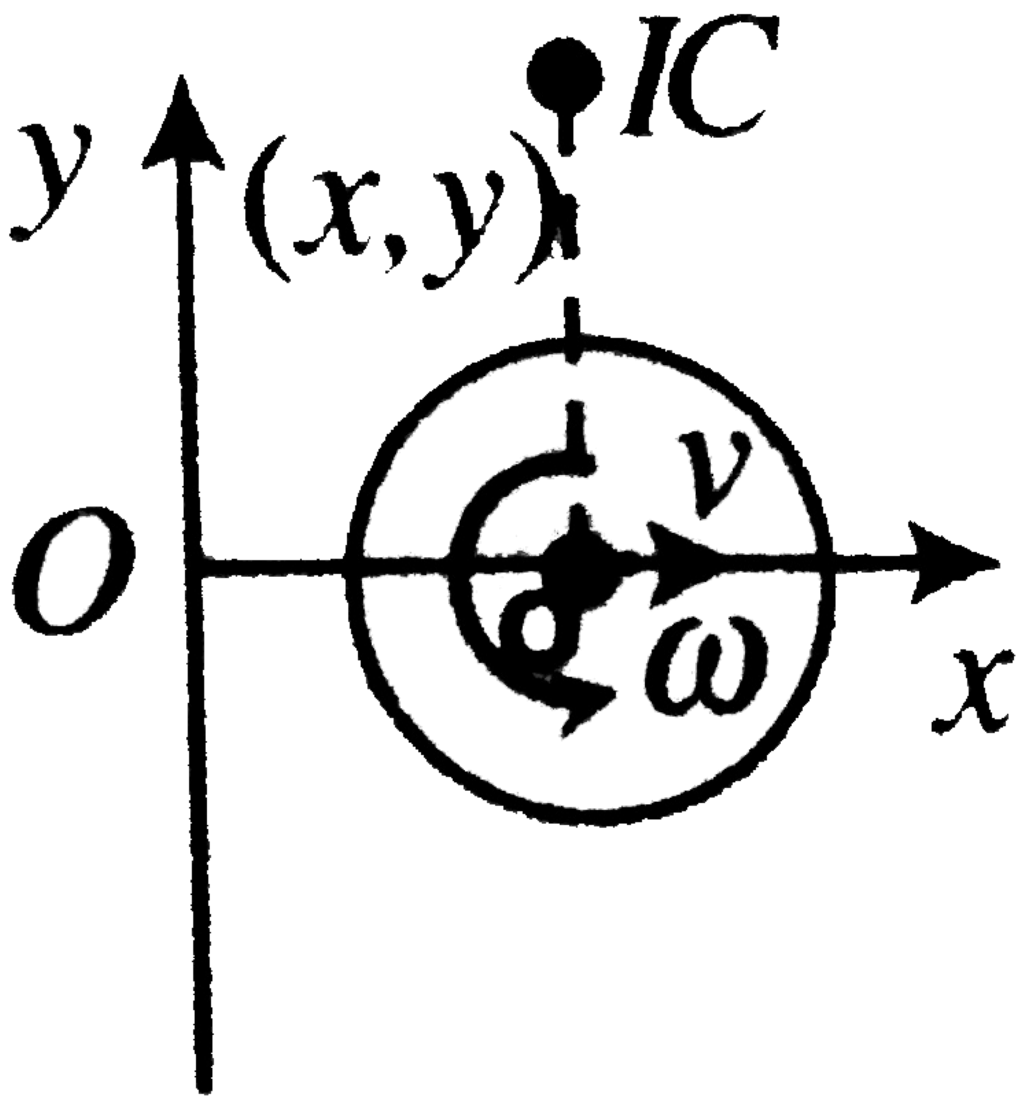
$\therefore y^2$

$= \left(\frac{2a}{\omega^2}\right) x (parabola)$

b. $v = \omega y$

$v = \frac{axy}{v}$

$\therefore xy = \frac{v^2}{\alpha} =$ constant (rectangular hyperbola)



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Q-16 - 17458928

A uniform cube of side and mass m rests on a rough horizontal surface. A horizontal force F is applied normal to one face at point that is directly above the centre of the face at a height $\frac{a}{4}$ above the centre. The minimum value of F for which the cube begins to topple above an edge without sliding is

(A) $\frac{1}{4}mg$

(B) 2mg

(C) $\frac{1}{2}mg$

(D) $\frac{2}{3}mg$

CORRECT ANSWER: D

Q-17 - 14928201

A billiard ball of mass m and radius r, when hit in a horizontal direction by a cue at a height h above its centre, acquired a linear velocity v_0. The angular velocity ω_0 acquired by the ball is

(A) $\dfrac{5v_0r^2}{2h}$

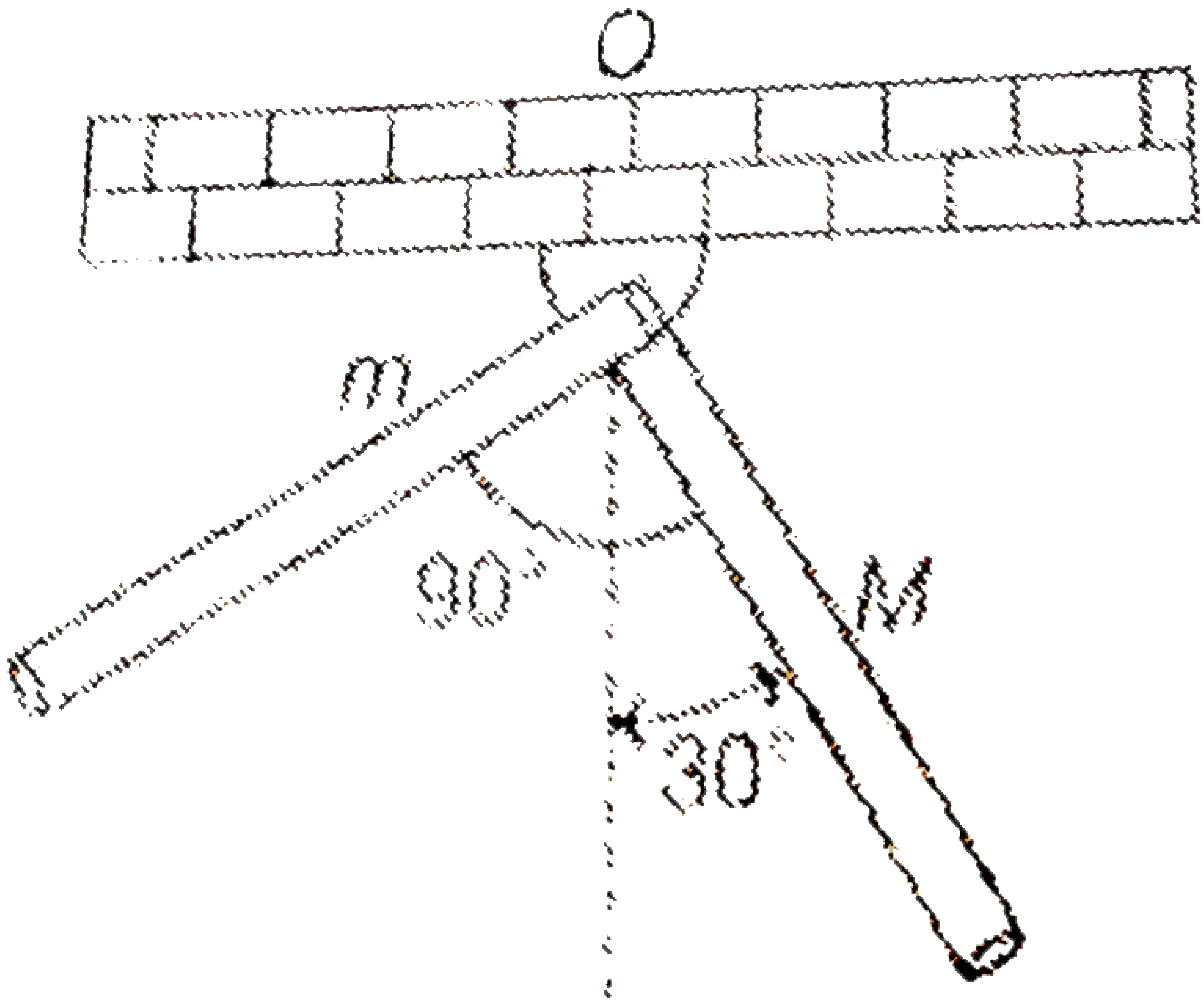
(B) $\dfrac{2v_0r^2}{5h}$

(C) $\dfrac{2v_0rh}{5r^2}$

(D) $\dfrac{5v_0h}{2r^2}$

Q-18 - 17458950

Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shwon. If in equilibrium, the body is in the shown configuration, ratio `M//m



will be

(A) 2

(B) 3

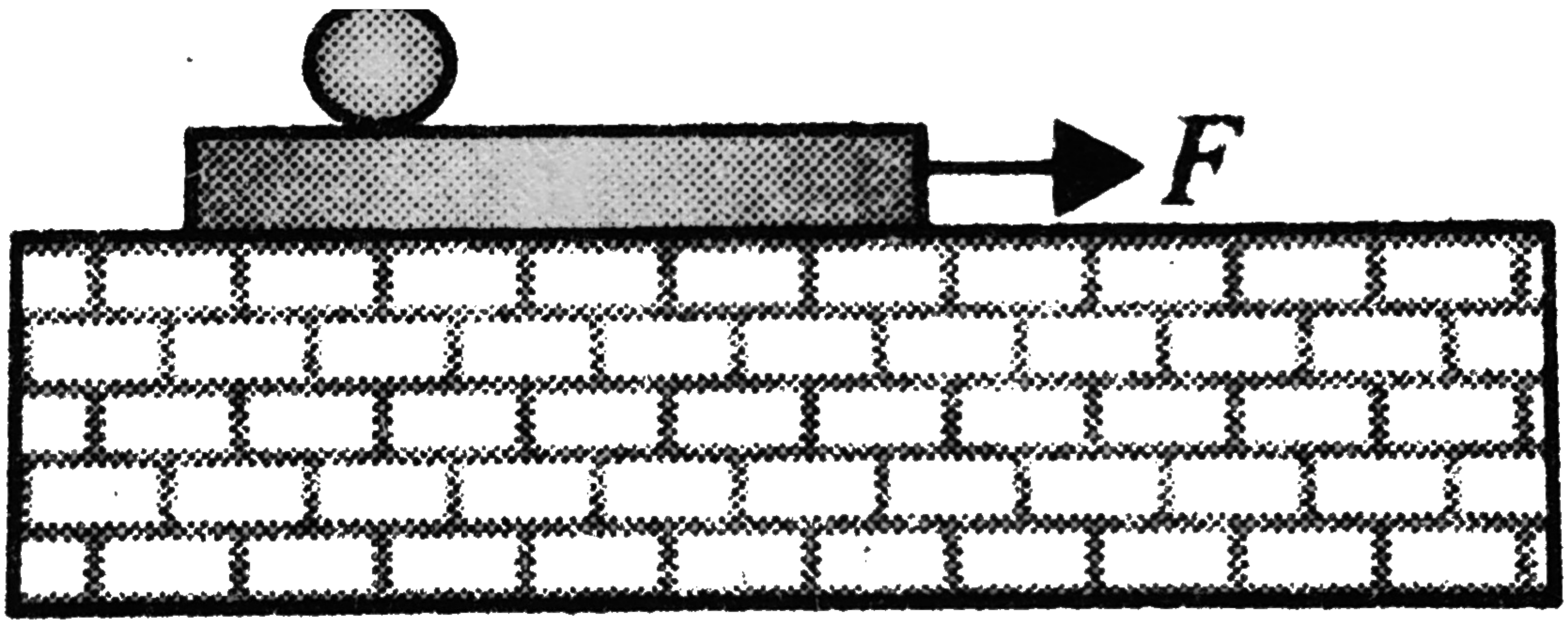
(C) $\sqrt{2}$

(D) $\sqrt{3}$

CORRECT ANSWER: D

Q-19 - 11301591

A plank with a uniform sphere placed on it resting on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is incorrect?



(A) acceleration of the centre of sphere is less than that of the plank

(B) work done by friction acting on the sphere is equal to its total kinetic energy

(C) total kinetic energy of the system is equal to work done by the force F

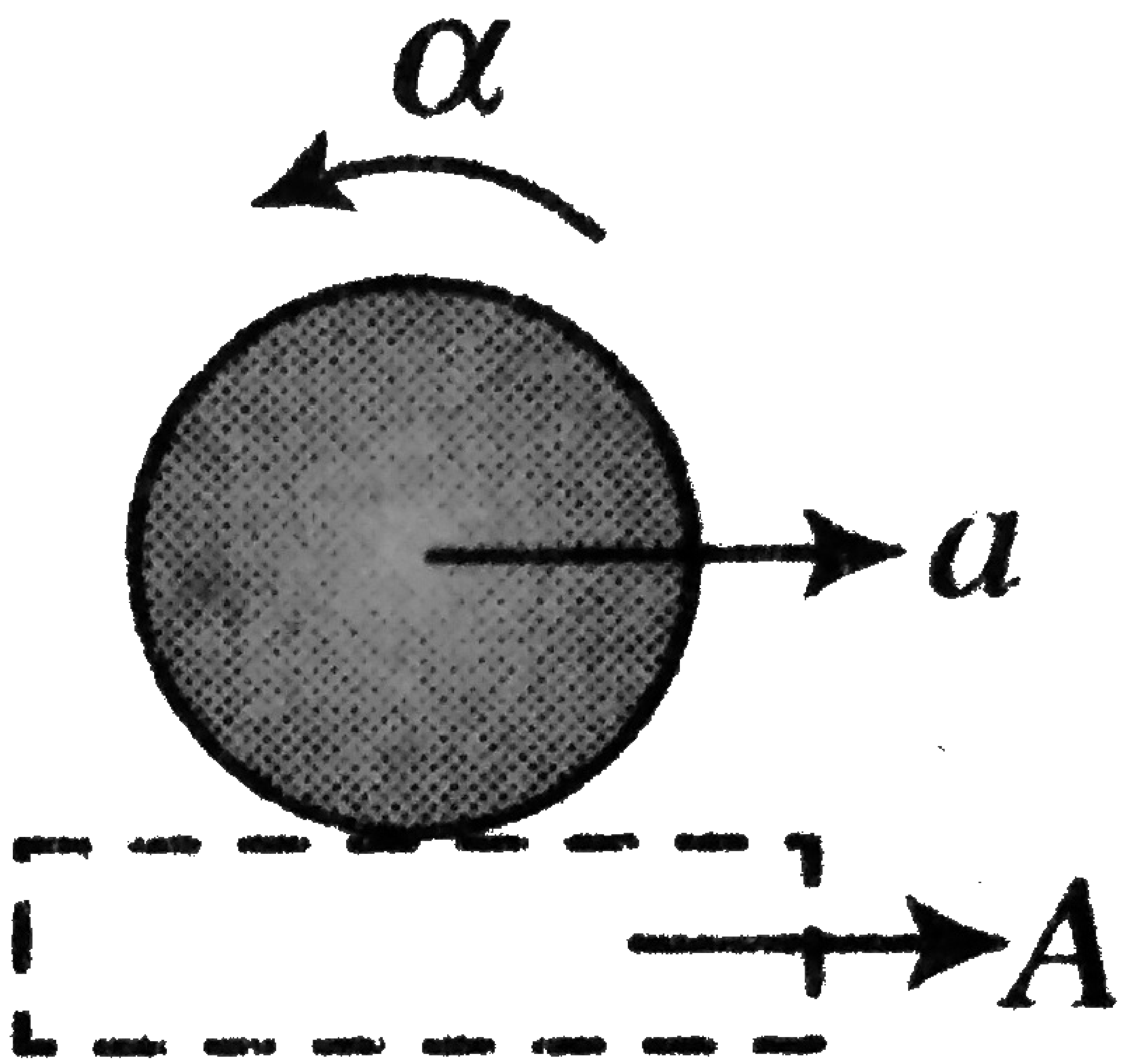
(D) none of the these

CORRECT ANSWER: D

SOLUTION:

From the shown figure $\alpha + R\alpha = A$

So $A > a$



By work energy theorem, work done by friction on sphere is change in its KE. If we consider sphere and plank as a system, then change in KE of plank and sphere is work done by F.

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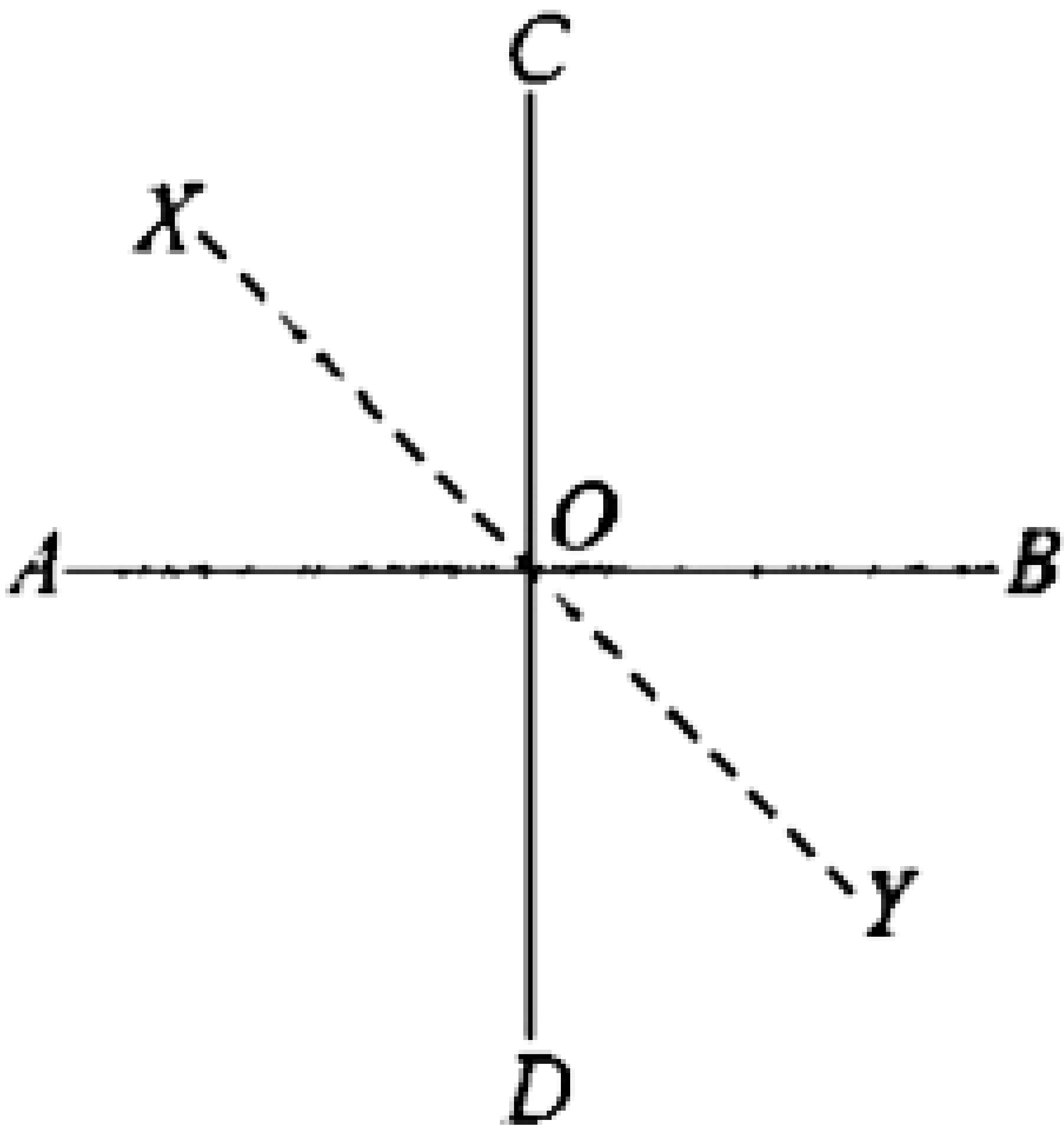
Q-20 - 17091909

AB and CD are two indential rods each of length L and mass M joined to from a cross. Find the M.L of the system about a bisector of the angel between the rods (XY):



(A) $\dfrac{ML^2}{12}$

(B) $\dfrac{ML^2}{6}$

(C) $\dfrac{ML^2}{3}$

(D) $\dfrac{4ML^2}{3}$

CORRECT ANSWER: A

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Q-21 - 13025828

The $M.I.$ of a rod about an axis through its center and perpendicular to it is I_0. The rod is bent in the middle so that the two halves make an angle θ. The moment of inertia of the bent rod about the same axis would be

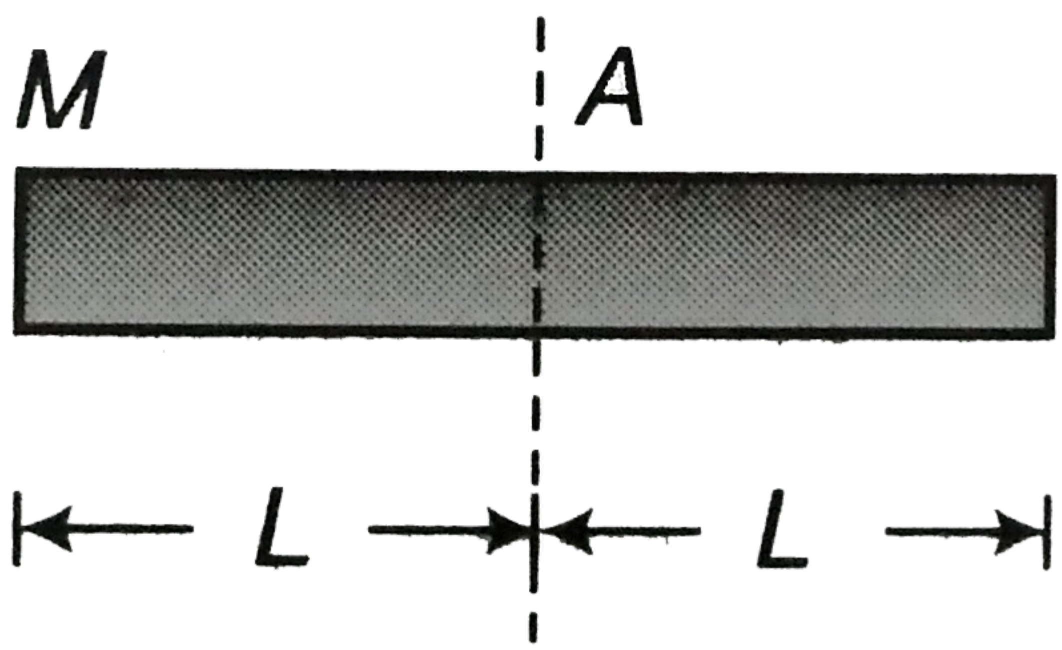
(A) $I_0 \sin^2 \theta$

(B) $I_0 \cos^2 \theta$

(C) I_0

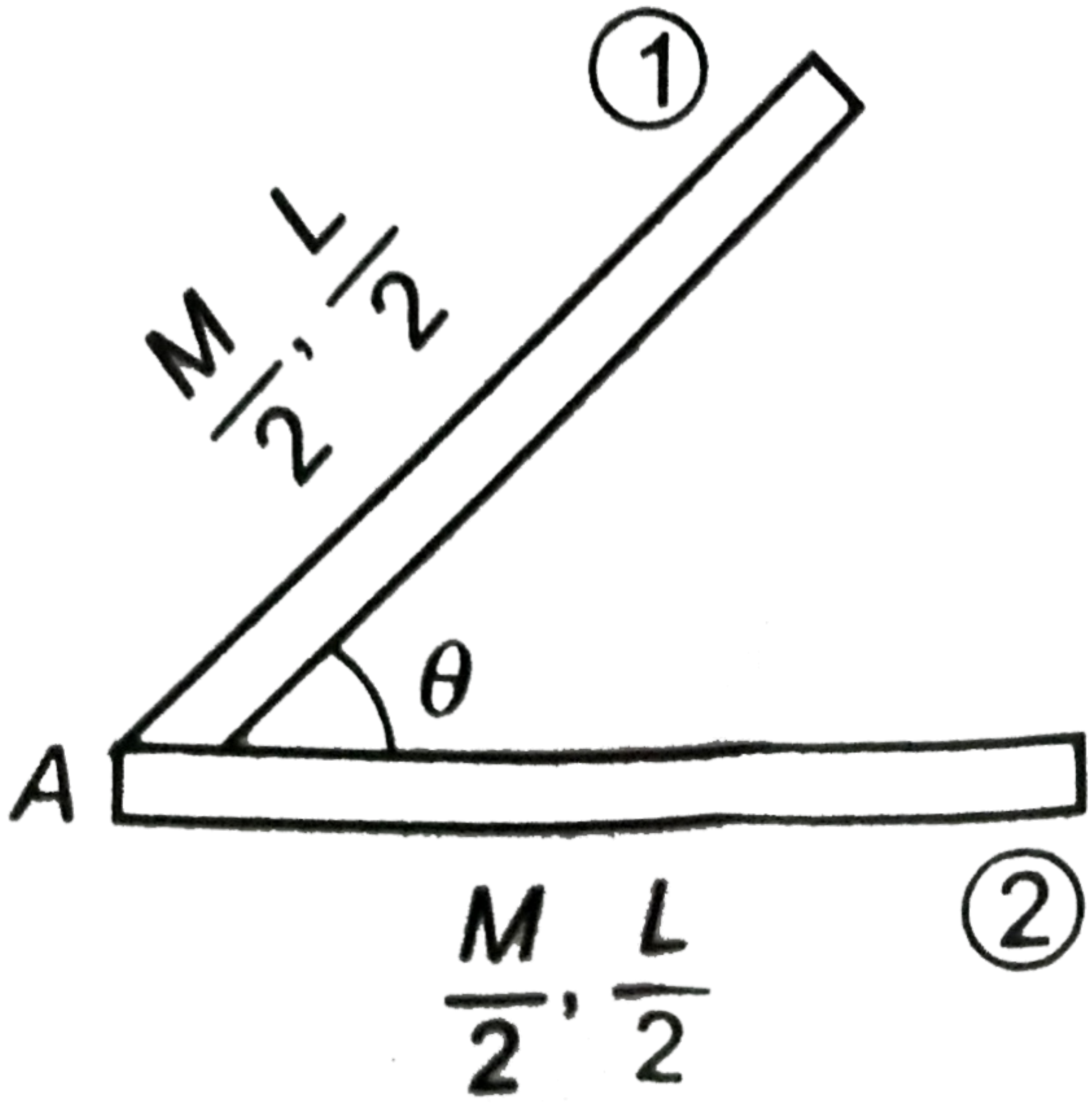
(D) $\frac{I_0}{2}$

SOLUTION:



$$I_A = \frac{ML^2}{12} = I_0$$

Now rod is bent at A



$M.I.$ of half rod (1) about A and $\perp^{ar}$ to the plane

$$I_1 = \frac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{24} = I_2$$

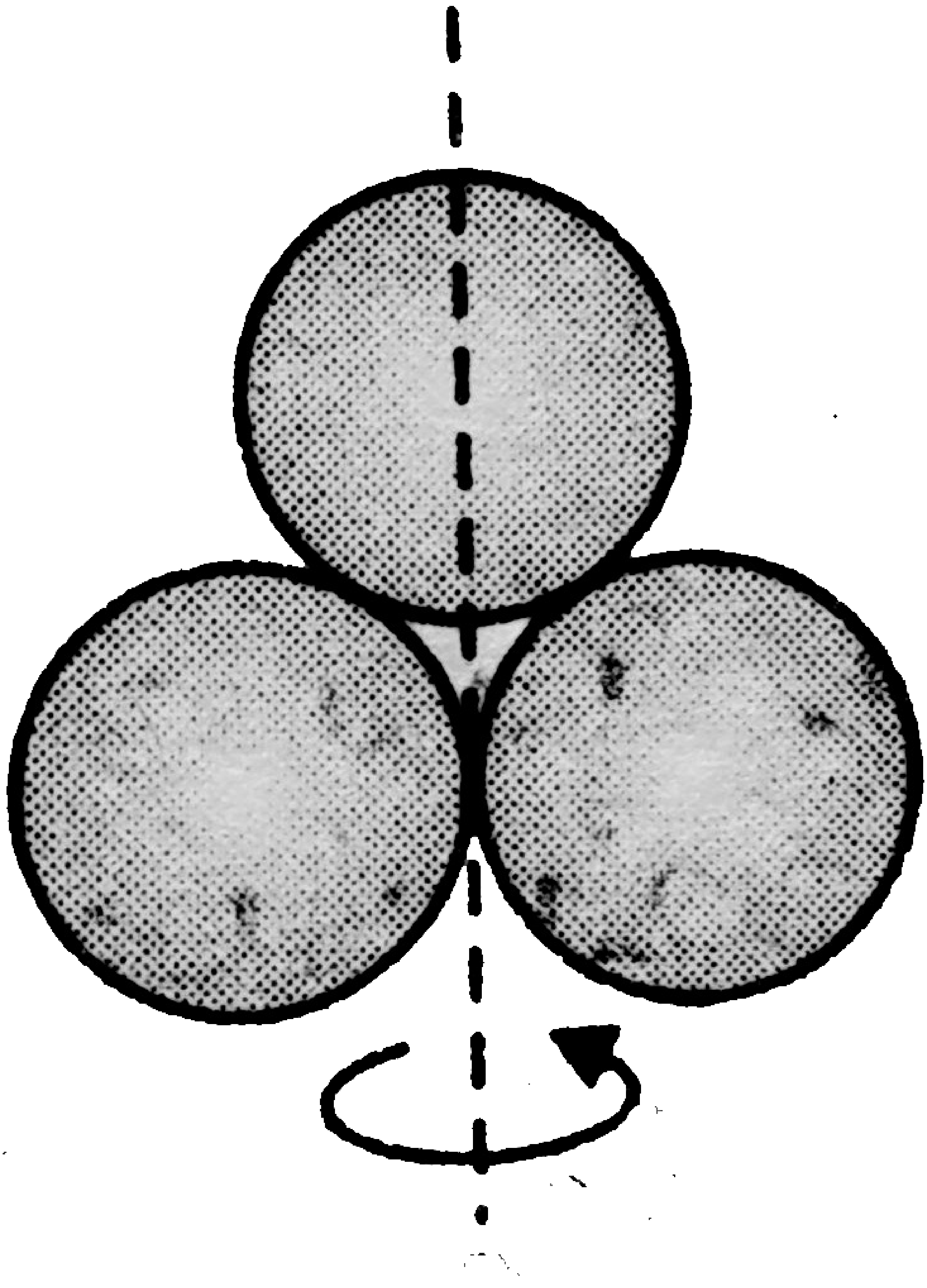
$$I_A = I_1 + I_2 = \frac{ML^2}{12}$$

$$= I_0$$

$M.I.$ will remain the same for any value of θ.

Q-22 - 11301175

Three rings, each of mass m and radius r, are so placed that they touch each other. Find the moment of inertia about the axis as shown in Fig.



(A) $5mr^2$

(B) $\frac{5}{7}mr^2$

(C) $7mr^2$

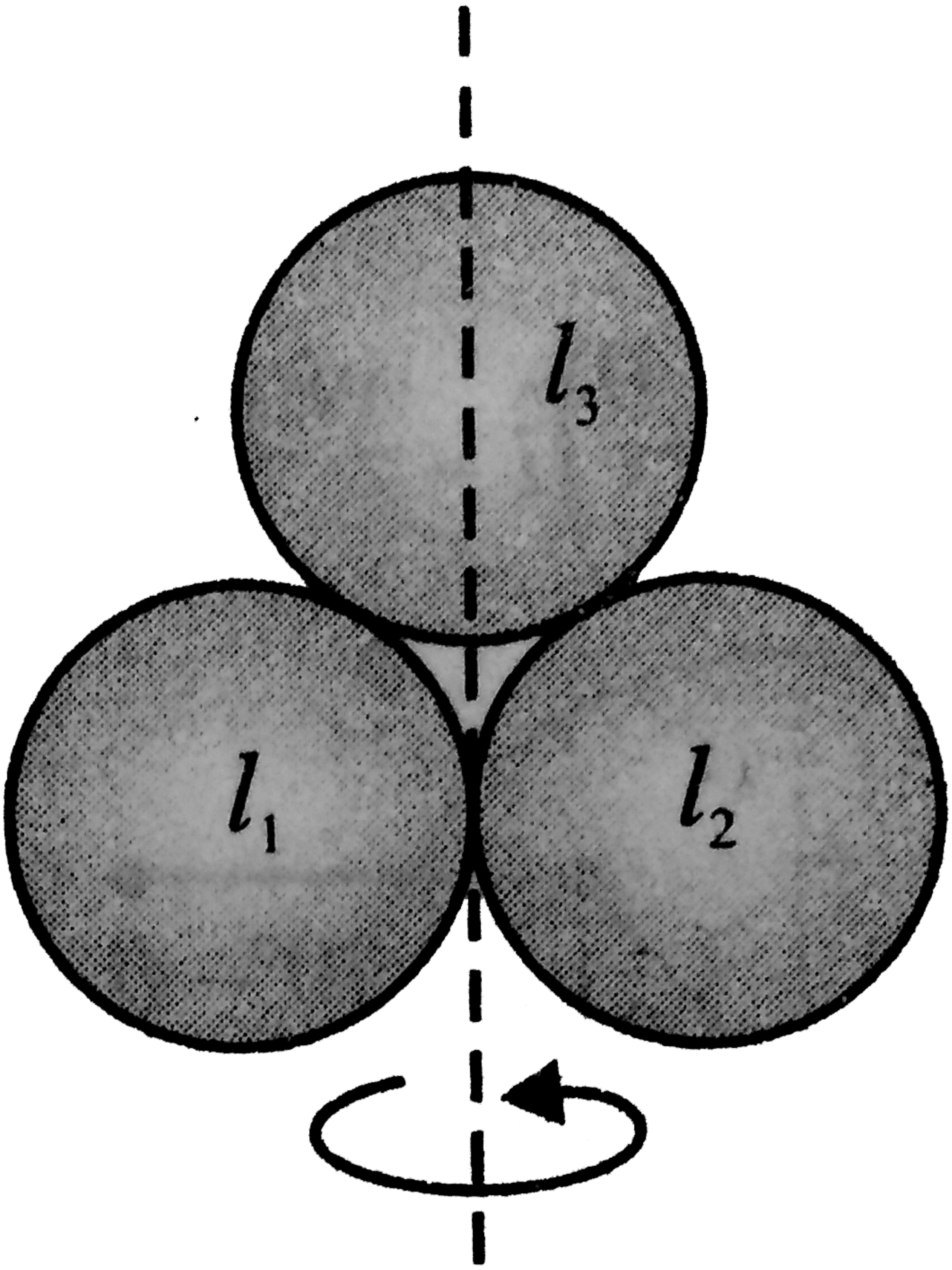
(D) $\frac{7}{2}mr^2$

CORRECT ANSWER: D

SOLUTION:

$$I = I_1 + I_2 + I_3$$

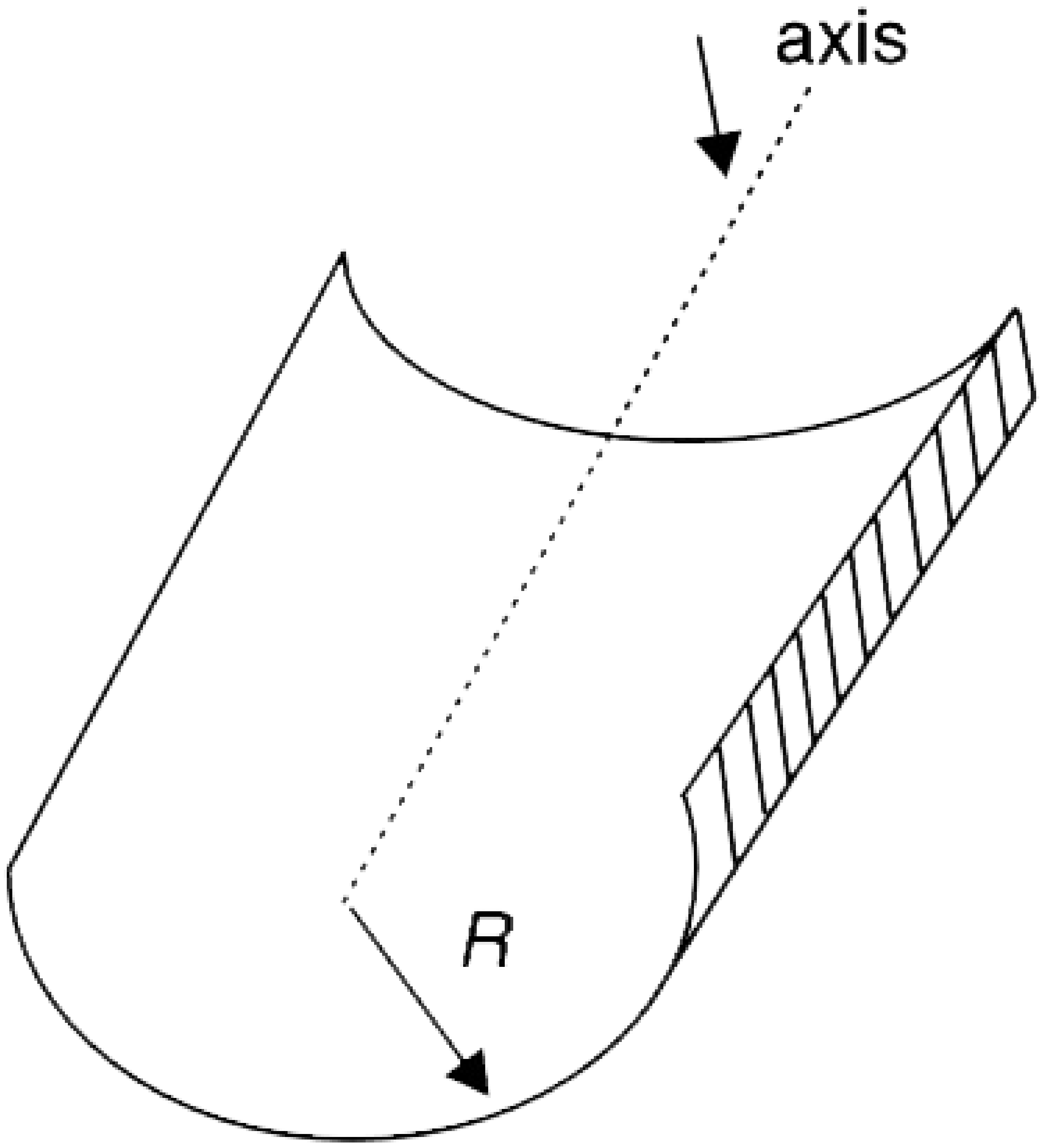
$$I_1 = I_2 = \frac{3}{2}mr^2$$



$$I_1 = I_2 = \frac{3}{2}mr^2$$
$$I = I_1 + I_2 + I_3$$
$$= \frac{7}{2}mr^2$$

Q-23 - 15159140

A thin semi circular cylindrical shell has mass M and radius R. Find its moment of inertia about a line passing through its centre of mass parallel to the axis (shown in figure) of the cylinder.



CORRECT ANSWER: $MR^2\left(1-\frac{4}{PI^2}\right)$

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Q-24 - 11765314

The moment of inertia of a uniform cylinder of length

l and $radius R$ about its perpendicular bisector is I. What is the

ratio l / R such that the moment of inertia is minimum ?

(A) 1

(B) $\frac{3}{\sqrt{2}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{\sqrt{3}}{2}$

CORRECT ANSWER: C

SOLUTION:

Moment of inertia of a uniform cylinder of length

l and $radius R$ about its perpendicular bisector is

$$I = \frac{ml^2}{12} + \frac{mR^2}{4}$$

$$= \frac{m}{4}\left(\frac{l^2}{3} + R^2\right)$$

..(i)

and $m = \pi R^2 l.\, \rho$

$$\therefore R^2 = \frac{m}{\pi / \rho}$$

From(i), $I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi l \rho}\right)$

I will be minimum, when

$$\frac{dI}{dl} = 0$$

$$\frac{d}{dl}\left[\frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi l \rho}\right)\right]$$

$$= 0$$

$$\frac{m}{4}\left(\frac{2l^2}{3} + \frac{m}{\pi l^2 \rho}\right)$$

$$= 0$$

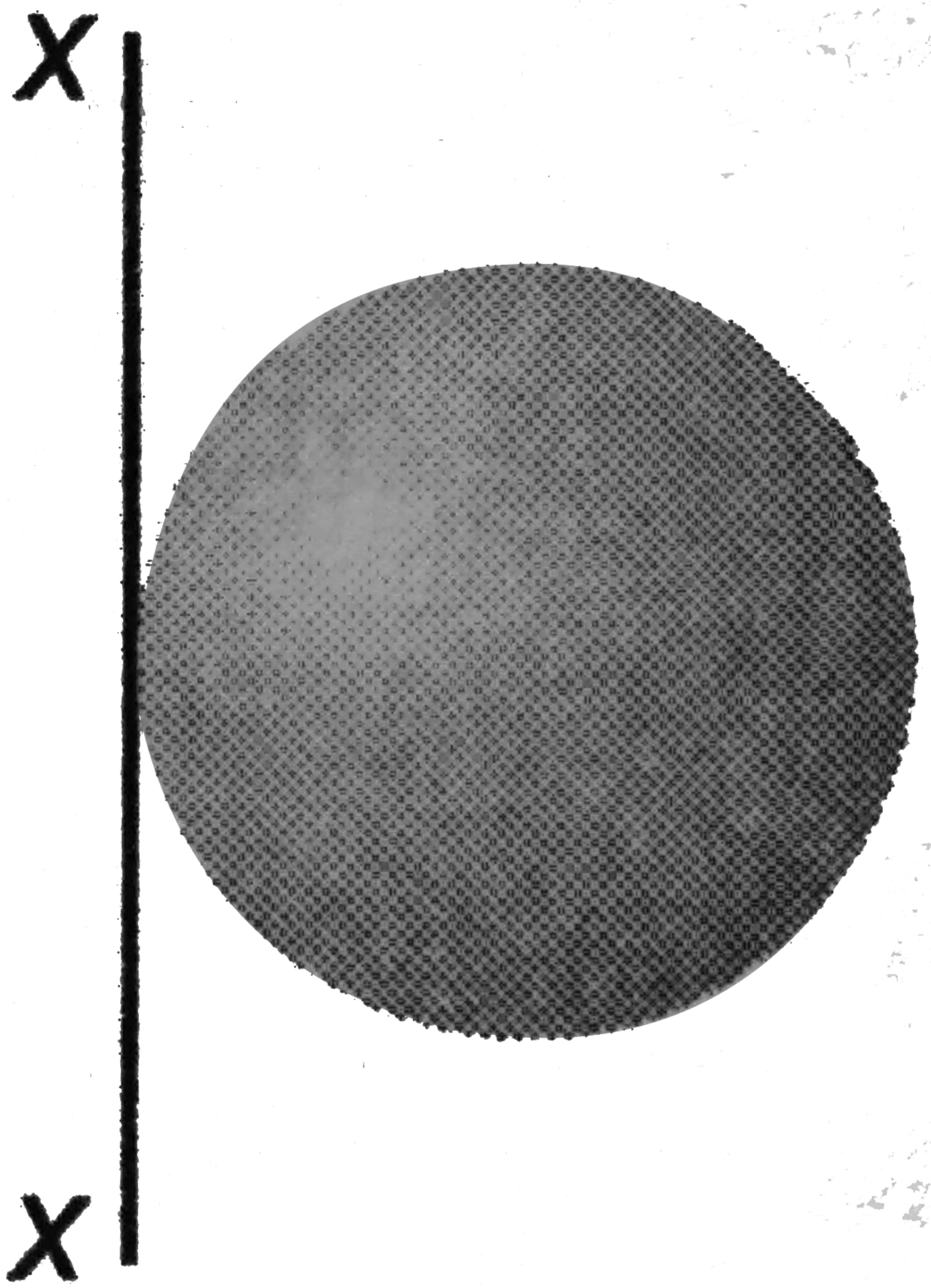
or

$$\frac{2l}{3} = \frac{m}{\pi l^2 \rho} = \frac{\pi R^2 l \rho}{\pi l^2 \rho}$$

$$= \frac{R^2}{l}$$

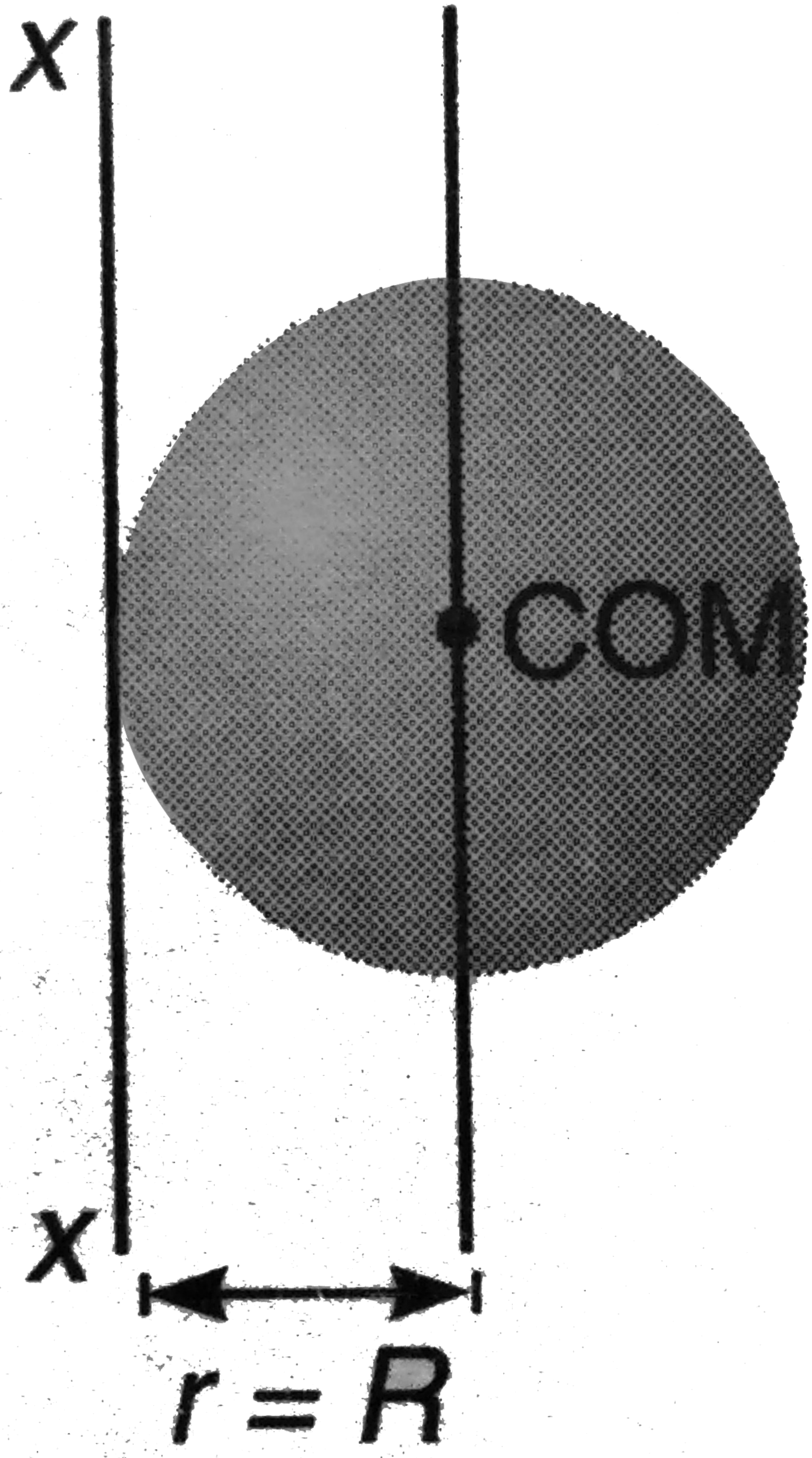
or $\frac{l^2}{R^2} = \frac{3}{2}, \frac{l}{R} = \sqrt{\frac{3}{2}}$

Q-25 - 10964020



Find the moment of inertia of a solid sphere of mass M and radias R about an axis XX shown in figure. Also find radius of gyration about the given axis.

SOLUTION:



From theorem of parallel axis

$$I_{XX} = I_{COM} + Mr^2$$
$$= \frac{2}{5}MR^2 + MR^2$$

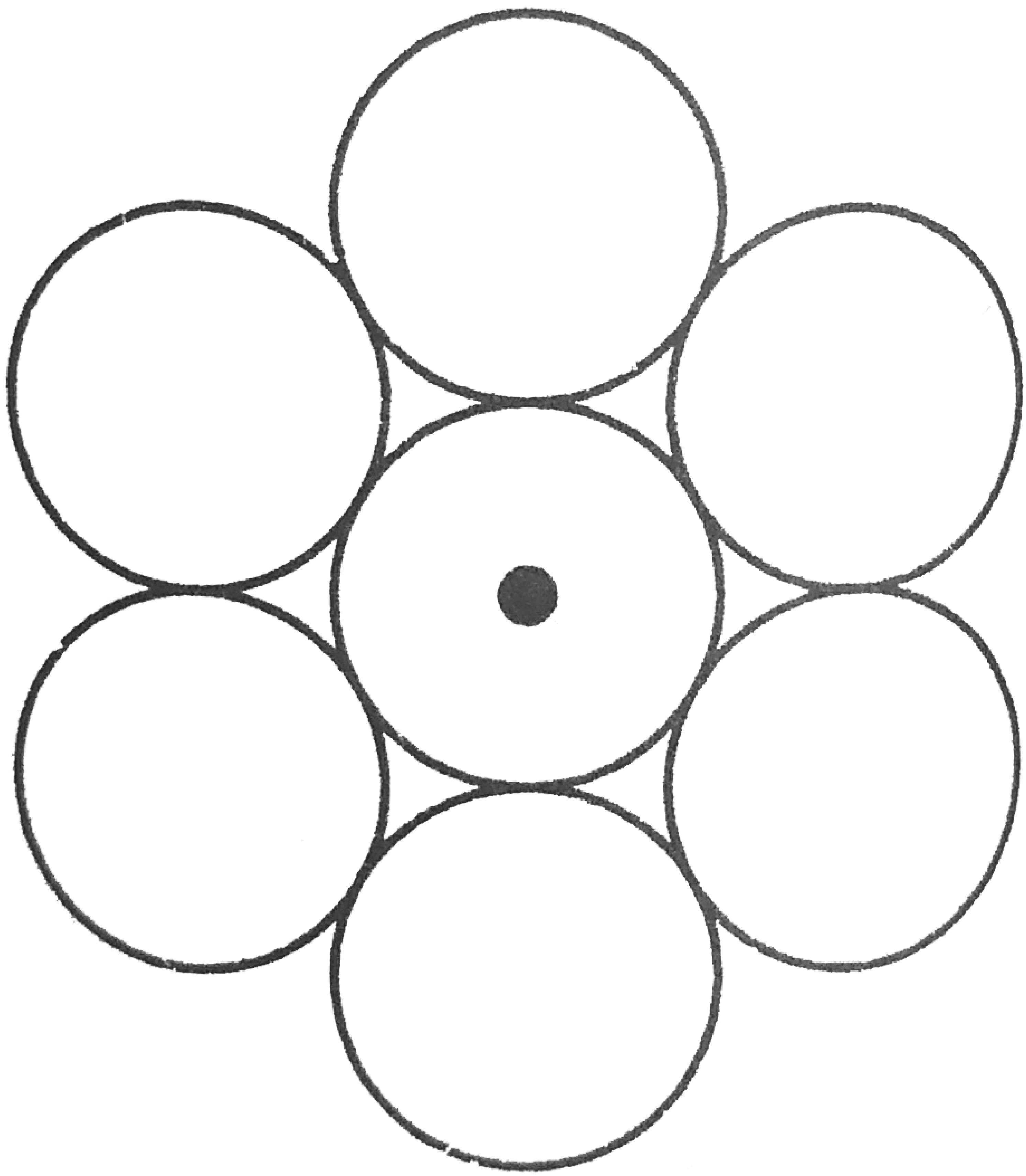
$$= \frac{7}{5}MR^2$$

Radius of gyration

$$K = \sqrt{\frac{I}{M}}$$
$$= \sqrt{\frac{\frac{7}{5}MR^2}{M}}$$
$$= \sqrt{\frac{7}{5}}R$$

Q-26 - 11748064

Seven identical disc are arranged in a hexagonal, planar pattern so as to touch each neighbour, as shown in the figure. Each disc has mass m and radius r. What is the moment of inertia of the system of seven disks about an axis passing through the centre of central disk and normal to plane of all disks ?



(A) $\frac{7}{2}mr^2$

(B) $\frac{13}{2}mr^2$

(C) $\frac{29}{2}mr^2$

(D) $\frac{55}{2}mr^2$

CORRECT ANSWER: D

SOLUTION:

(d) $I = I_0 + 6I'$

I_0 is the moment of inertia of central disc and I' is moment of inertia of rest of the each disc about specified axis.

By parallel axes theorem

$$I = \frac{mr^2}{2} + 6\left(I_{cm} + md^2\right)$$

$$= \frac{mr^2}{2} + 6\left[\frac{mr^2}{2} + m(2r)^2\right]$$

=

$$\frac{mr^2}{2} + \frac{6mr^2}{2} + 24mr^2 = \frac{55mr^2}{2}$$

.

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Q-27 - 15159141

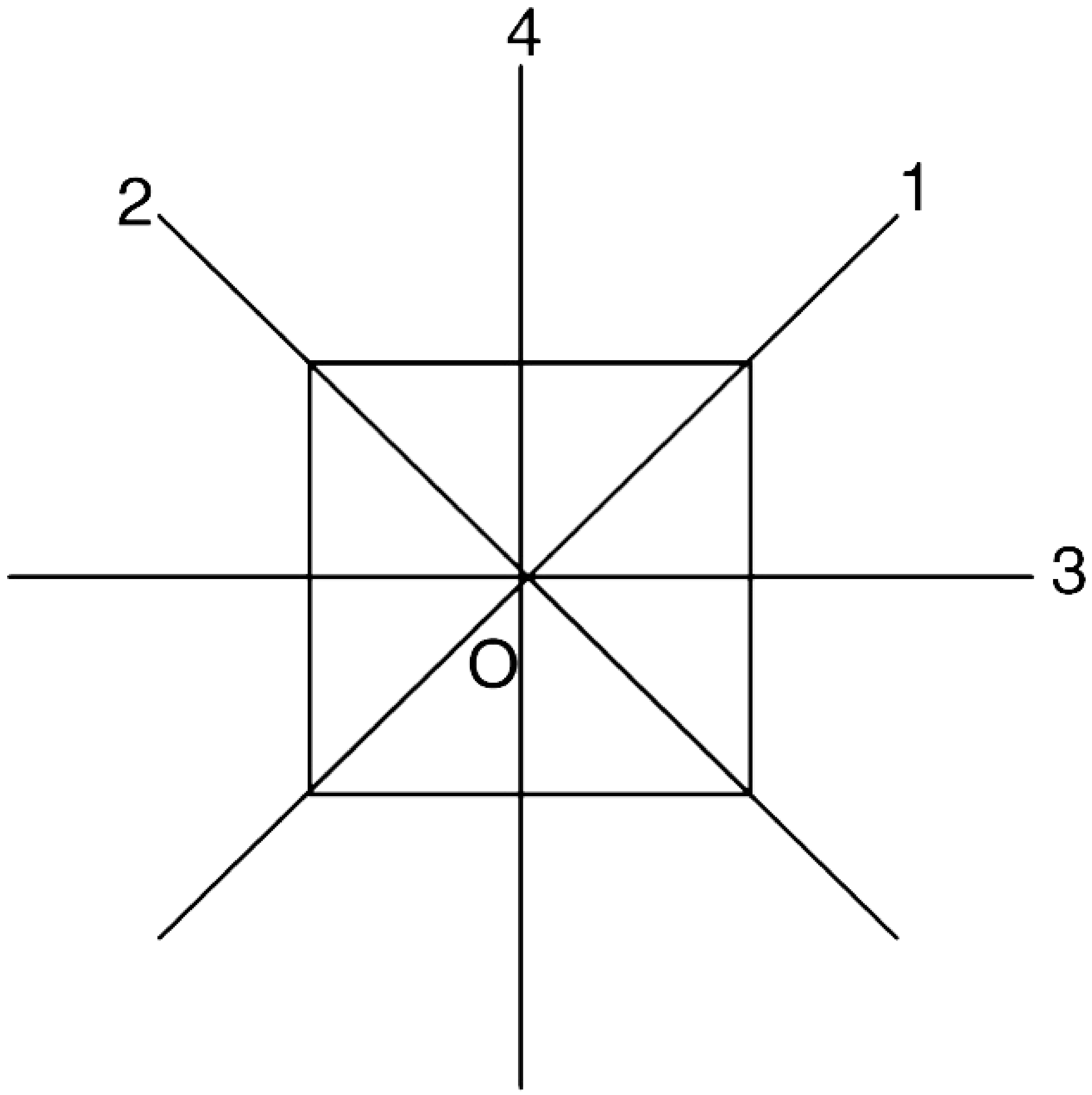
Consider a uniform square plate shown in the figure. I_1, I_2, I_3 and I_4 are moment of inertia of the plate about the axes $1, 2, 3$ and 4 respectively. Axes 1 and 2 are diagonals and 3 and 4 are lines passing through centre parallel to sides of the square. The moment of inertia of the plate about an axis passing through centre and perpendicular to the plane of the figure is equal to which of the followings.

(a) $I_3 + I_4$

(b) $I_1 + I_3$

(c) $I_2 + I_3$

(d) $\frac{1}{2}(I_1 + I_2 + I_3 + I_4)$



CORRECT ANSWER: A, B, C, D

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Q-28 - 13025826

The moment of inertia of a uniform rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. Then I_1 / I_2 is

(A) $\dfrac{\pi^2}{3}$

(B) $\dfrac{2\pi^2}{3}$

(C) $\dfrac{4\pi^2}{3}$

(D) $\dfrac{8\pi^2}{3}$

SOLUTION:

$$I_1 = \frac{ML^2}{3}$$

$$L = 2\pi R \Rightarrow R = \frac{L}{2\pi}$$

$$I_2 = \frac{1}{2}MR^2$$

$$= \frac{1}{2}M\left(\frac{L}{2\pi}\right)^2$$

$$= \frac{ML^2}{8\pi^2}$$

$$\frac{I_1}{I_2} = \frac{8\pi^2}{3}$$

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Q-29 - 11765313

An autmobile moves on road with a speed of $54km/h$. The radius of its wheel is $0.45m$ and the moment of inertia of the wheel about its axis of rotation is $3kgm^2$. If the vehicle is brought to rest in $15s$, the magnitude of average torque tansmitted by its brakes to the wheel is :

(A) $2.86kgm^2s^{-2}$

(B) $6.66kgm^2s^{-2}$

(C) $8.58kgm^2s^{-2}$

(D) $10.86kgm^2s^{-2}$

CORRECT ANSWER: B

SOLUTION:

Here,

$u = 54km / h = 15m$

$/ s, r = 0.45m$

,

$I = 3kgm^2, t$

$= 15\sec, \omega = 0$

Now

$u = r\omega_0 \text{ or } \omega_0 = \frac{u}{r}$

$= \frac{15}{0.45} = \frac{100}{3}$

As $\omega = \omega_0 + \alpha t$

$0 = \frac{100}{3} + (-\alpha)(15)$ [As force is retarding]

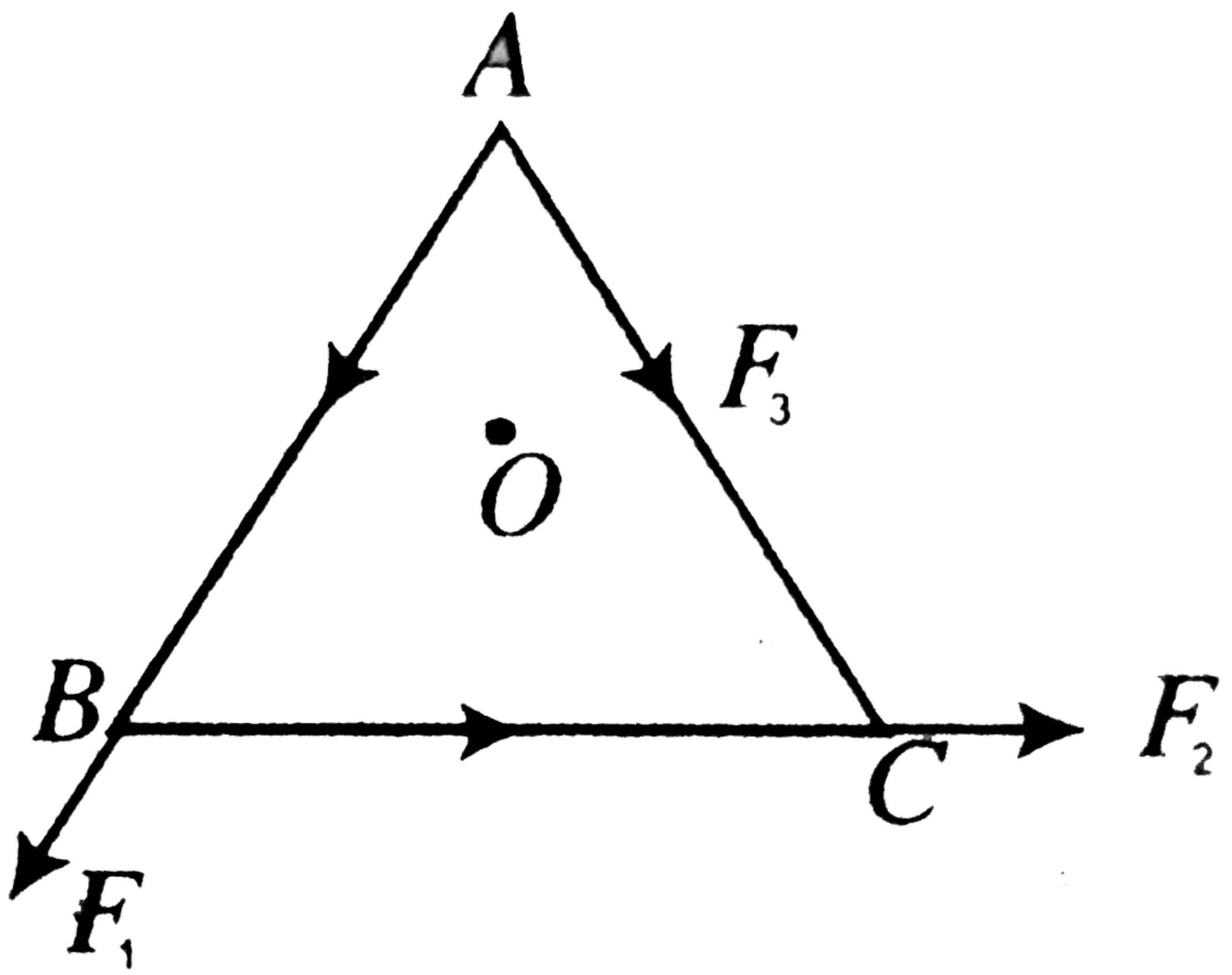
or $\alpha = \frac{100}{54}$, Also $\tau = I\alpha$

$= 3 \times \frac{100}{45}$

$= 6.66kgm^2 s^{-2}$

Q-30 - 11301243

O is the centre of an equilateral triangle ABC. F_1, F_2 and F_3 are the three forces acting along the sides AB, BC and AC respectively. What should be the value of F_3 so that the total torque about O is zero?



(A) $2(F_1 + F_2)$

(B) $\dfrac{F_1 + F_2}{2}$

(C) $F_1 - F_2$

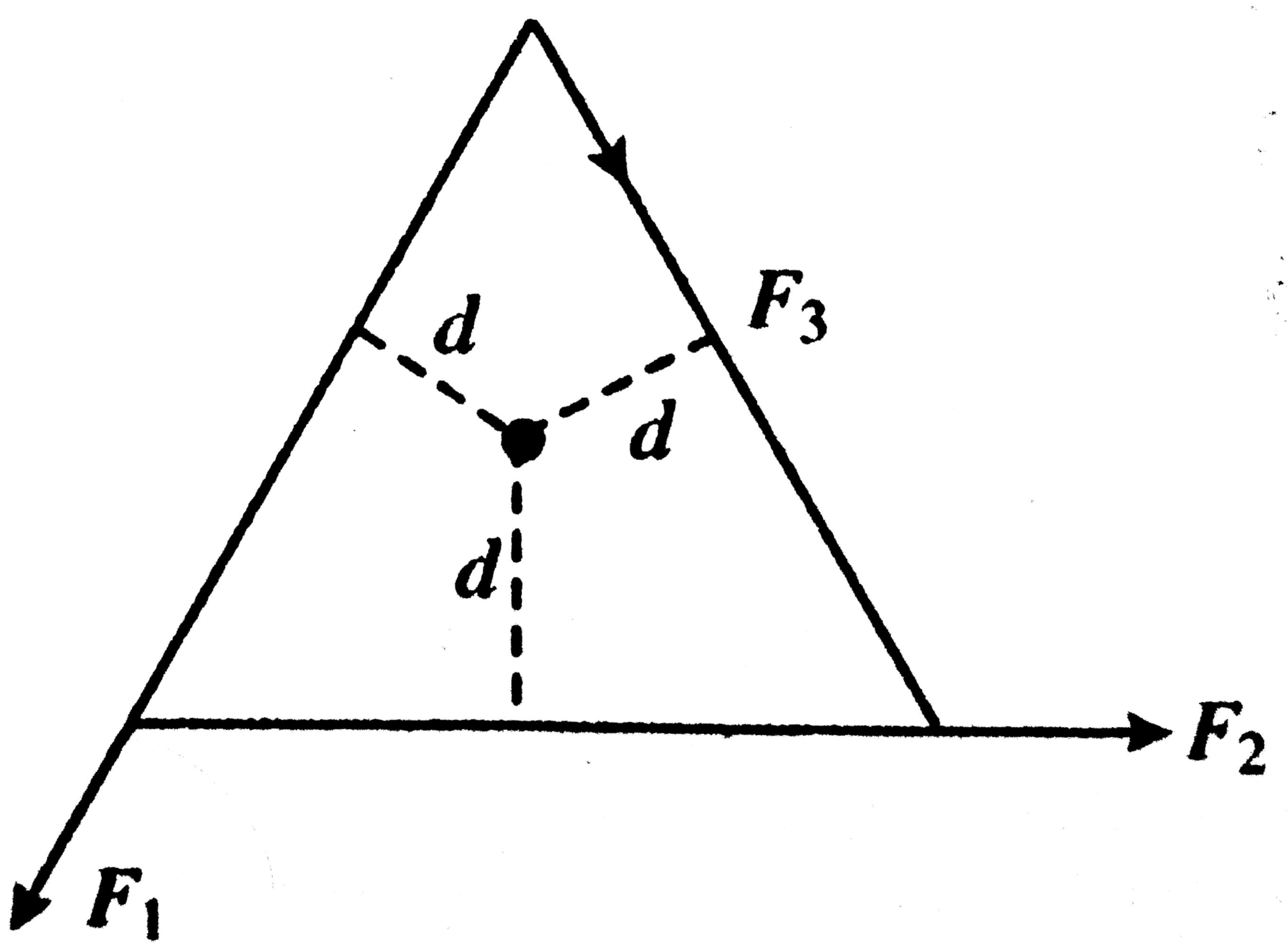
(D) $F_1 + F_2$

CORRECT ANSWER: D

SOLUTION:

The torquues $F_1 d$ and $F_2 d$ of F_1 and F_2 respectively are counterclockwise. The torque $F_3 d$ is clockwise.

Applying condition for rotational equilibrium



$F_1 d + F_2 d - F_3 d = 0$ or $F_3 = F_1 + F_2$

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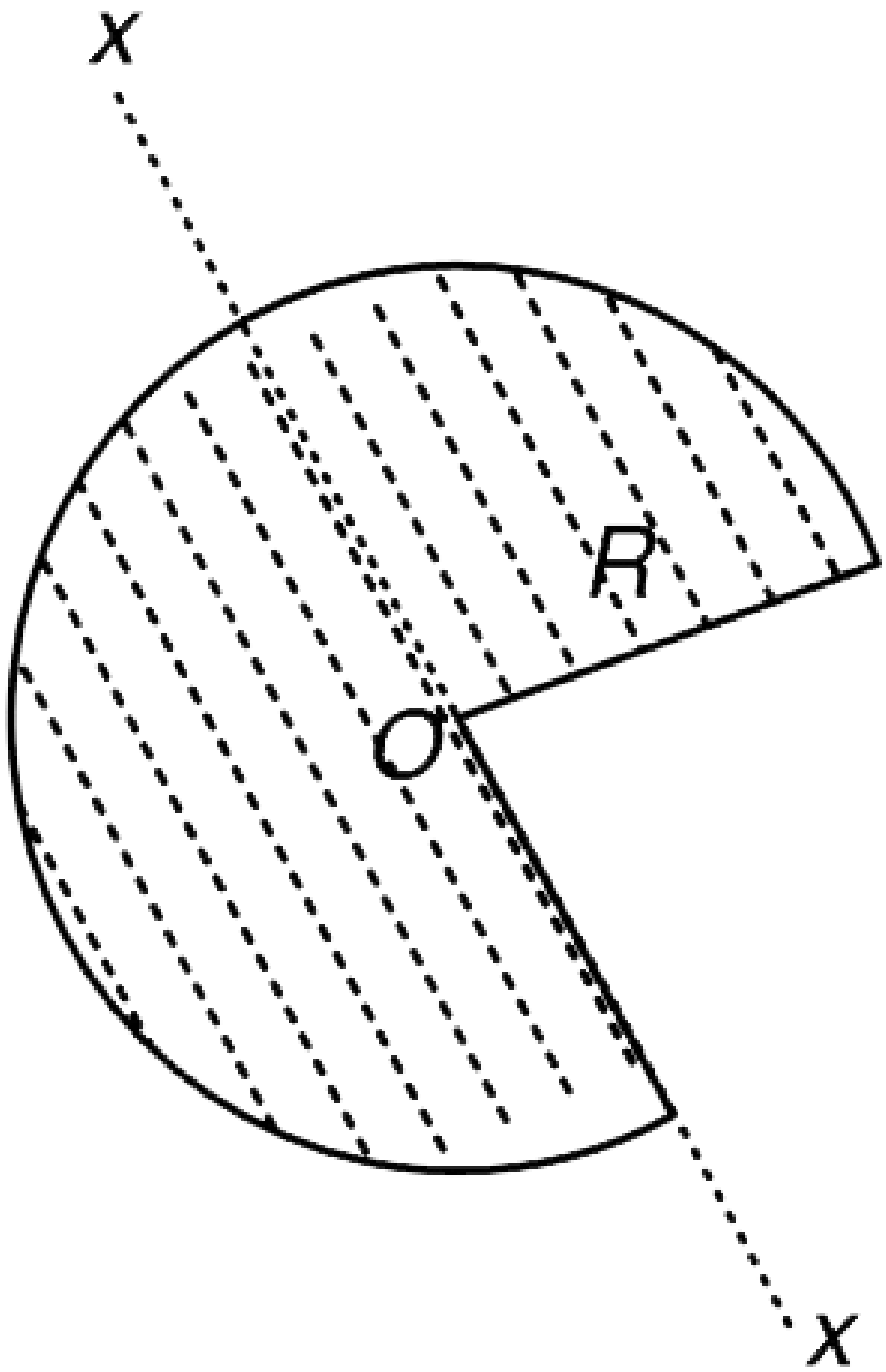
Q-31 - 15159136

A uniform circular disc has a sector of angle 90 removed from it. Mass of the remaining disc is M. Write the moment of inertia of the remaining disc about the axis xx shown in figure (Radius is R)



CORRECT ANSWER: $\frac{1}{4}MR^2$

Q-32 - 13076442

M is mass and R is radius of a circular ring. The moment of inertia of same ring about an axis in the plane of ring at a perpendicular distance $2\frac{R}{3}$ from centre of ring is

(A) $\frac{2MR^2}{3}$

(B) $\frac{4MR^2}{9}$

(C) $\frac{3MR^2}{8}$

(D) $\frac{17MR^2}{18}$

CORRECT ANSWER: D

SOLUTION:

$I = I_c = \; + Md^2, I_C$

$= MR^2$

and $d = \frac{2R}{3}$

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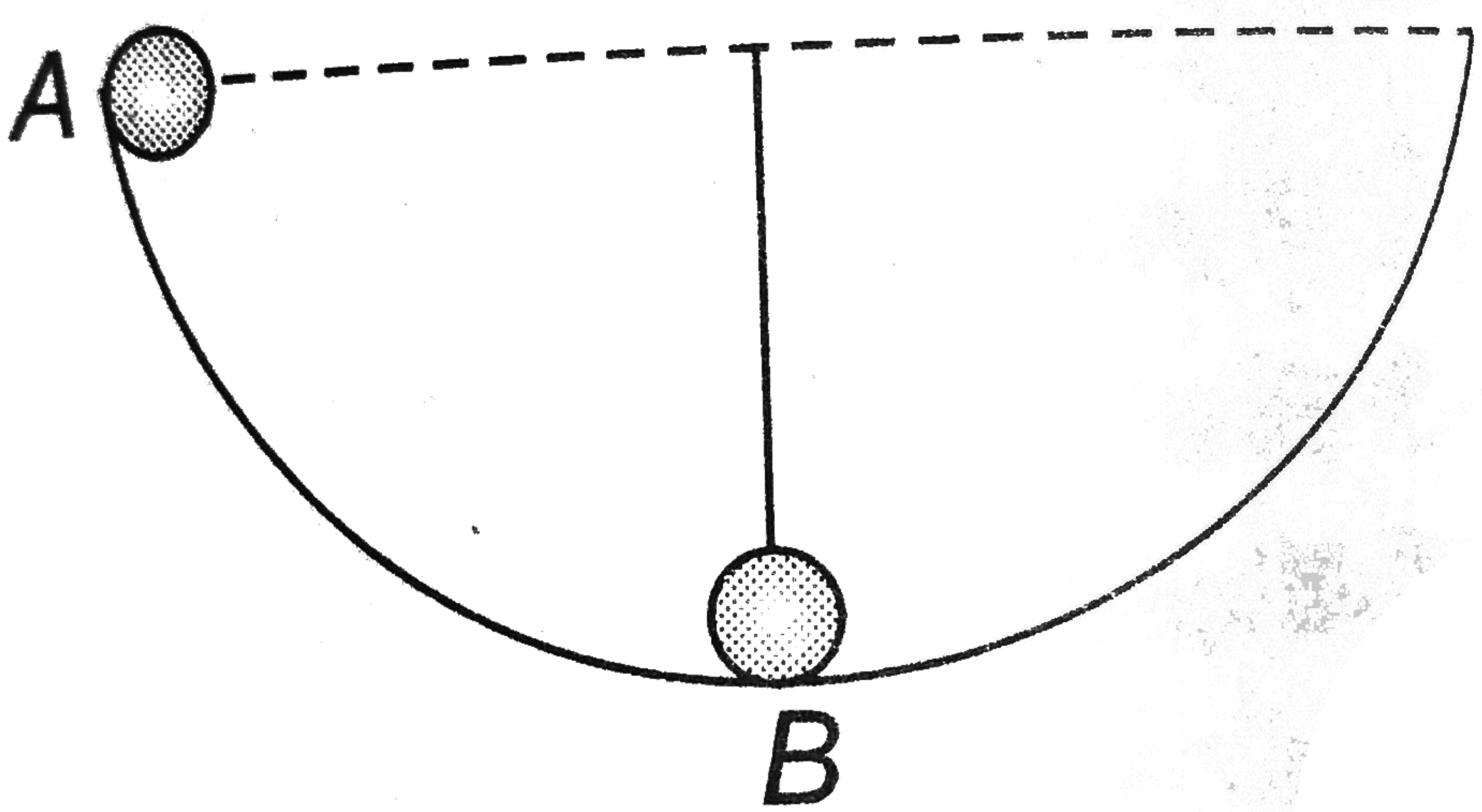
Q-33 - 11748366

A ball of mass m and radius r rolls inside a hemispherical shell of radius R. It is released from rest from point A as shown in figure. The angular velocity of centre of the ball in position B about the centre of the shell is.



(A) $2\sqrt{\frac{g}{5(R-r)}}$

(B) $2\sqrt{\dfrac{g}{7(R-r)}}$

(C) $\sqrt{\dfrac{g}{7(R-r)}}$

(D) $\sqrt{\dfrac{5g}{2(R-r)}}$

CORRECT ANSWER: B

SOLUTION:

(b) $K.E.$ of ball in position $B = mg(R-r)$

Here m = mass of ball

Since it rolls without slipping the ratio of rotational to

translational kinetic energy will be $2/5$.

$$\frac{K_R}{K_T} = \frac{2}{5}$$
$$K_T = \frac{2}{7}mg(R-r)$$
$$\frac{1}{2}mv^2 = \frac{2}{7}mg(R-r)$$
$$v = \frac{2}{\sqrt{7}}\sqrt{g(R-r)}$$

$$\omega = \frac{v}{R - r}$$

$$= 2\sqrt{\frac{g}{7(R - r)}}$$

.

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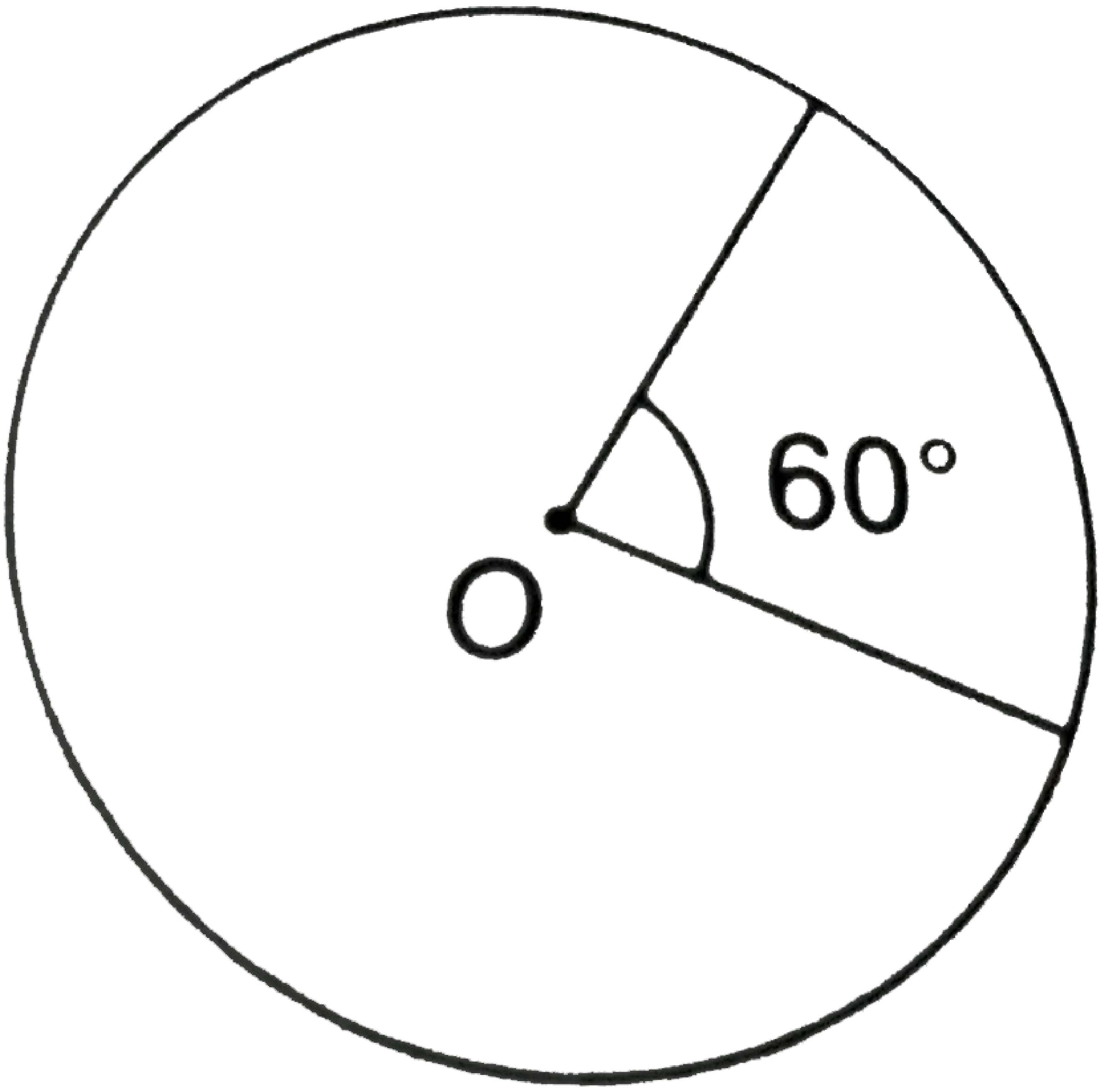
Q-34 - 13025839

From a circular disc of mass M and radius R, a part of 60 is removed. The $M.I.$ of the remaining portion of disc about an axis passing through the center and perpendicular to plane of disc is



(A) $\frac{5}{6}MR^2$

(B) $\frac{5}{3}MR^2$

(C) $\frac{5}{12}MR^2$

(D) $\frac{5}{24}MR^2$

SOLUTION:

Mass of the removed portion $= M \times \frac{60}{360} = \frac{M}{6}$

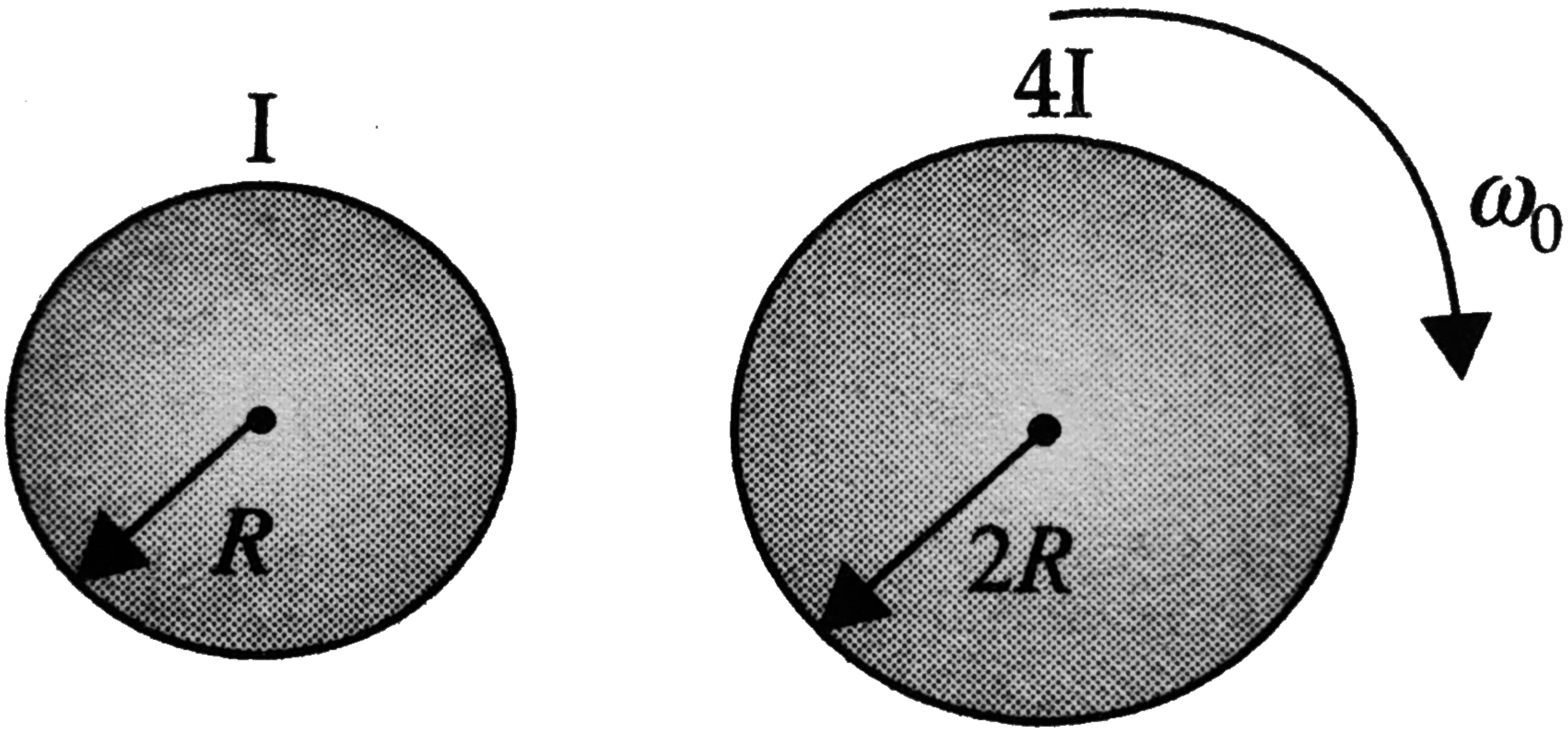
Its $M.I.\ I_1 = \frac{1}{2}\frac{M}{6}R^2$

$M.I.$ of the complete disc $I_2 = \frac{1}{2}MR^2$

$M.I.$ of the remaining portion $= I_2 - I_1 = \frac{5}{12}MR^2$

Q-35 - 11301611

Two cylinders having radii $2R$ and R and moment of inertia $4I$ and I about their central axes are supported by axles perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is caused to rotate by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. During this



(A) angular momentum of system is conserved

(B) kinetic energy is conserved

(C) neither the angular momentum nor the kinetic energy is conserved

(D) both the angular momentum and kinetic energy are conserved

---

CORRECT ANSWER: C

SOLUTION:

Angular momentum of system can not remain consrved as some external unbalanced torque is present due to forces at axels. Kinetic energy is not conserved, because slipping is there and work is done against friction.

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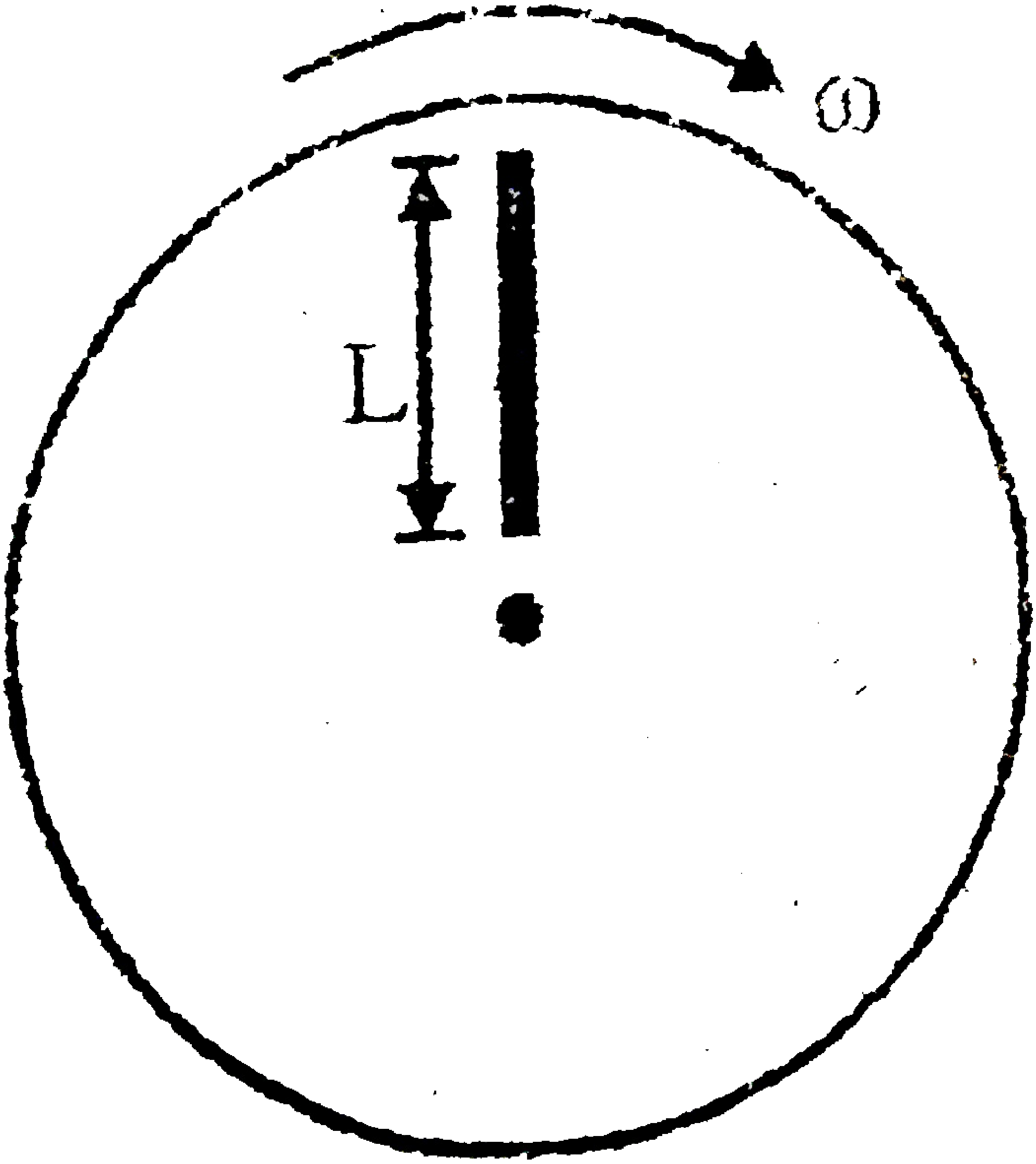
Q-36 - 15217343

A uniform rod of mass m and length L lies radialy on a disc rotating with angular speed ω in a horizontal plane about vertical axis passing thorugh centre of disc. The rod does not slip on the disc and the centre of the rod is at a distance 2L from the centre of the disc. them the kinetic energy of the rod is



(A) $\dfrac{49mL^2\omega^2}{24}$

(B) $2m\omega^2L^2$

(C) $\dfrac{1}{24}m\omega^2L^2$

(D) $\dfrac{13mL^2\omega^2}{24}$

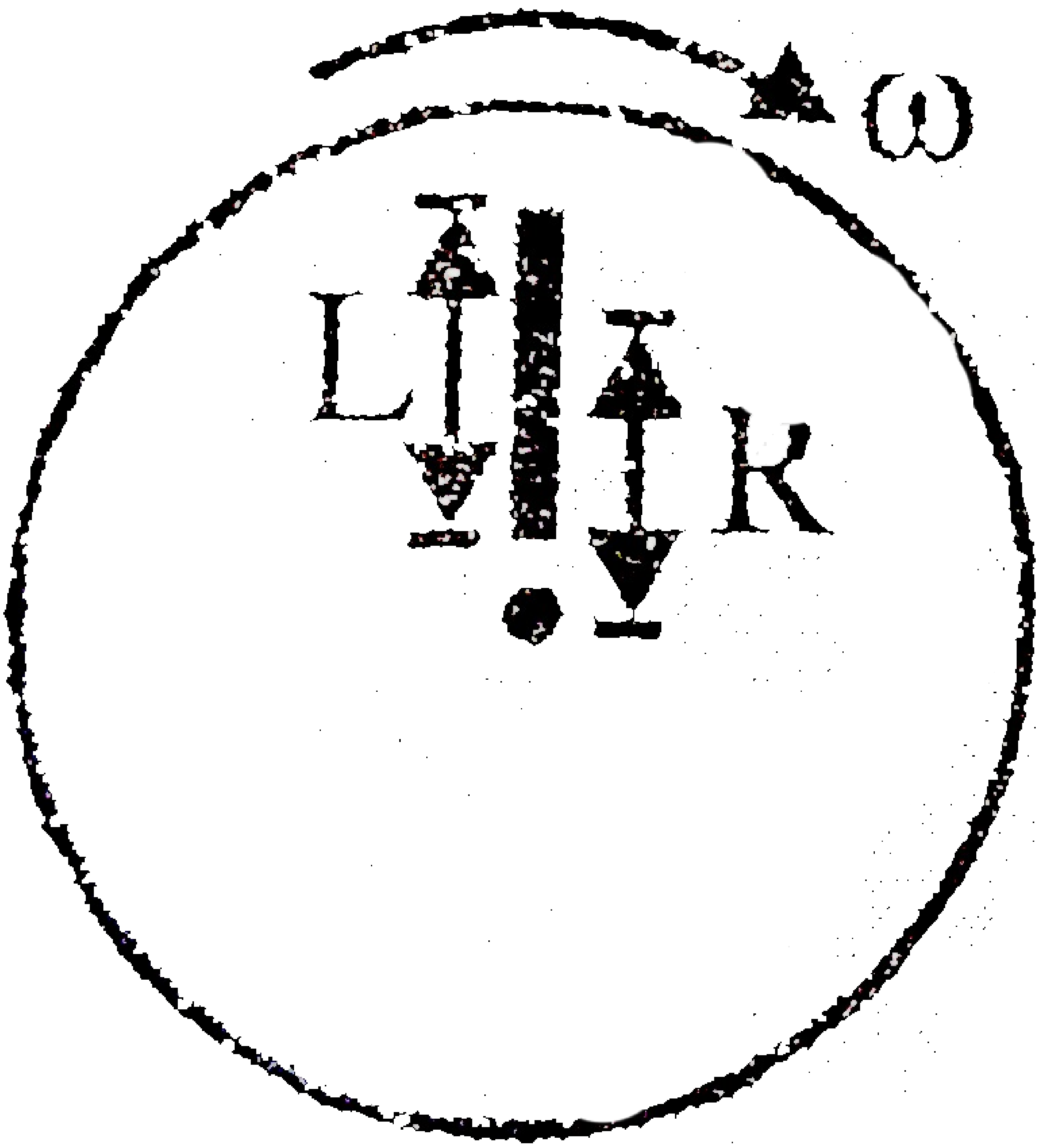
CORRECT ANSWER: 4

SOLUTION:

Moment of inertia of the rod w.r.t the axis through centre

of the disc si : (by parallel axis theoram)

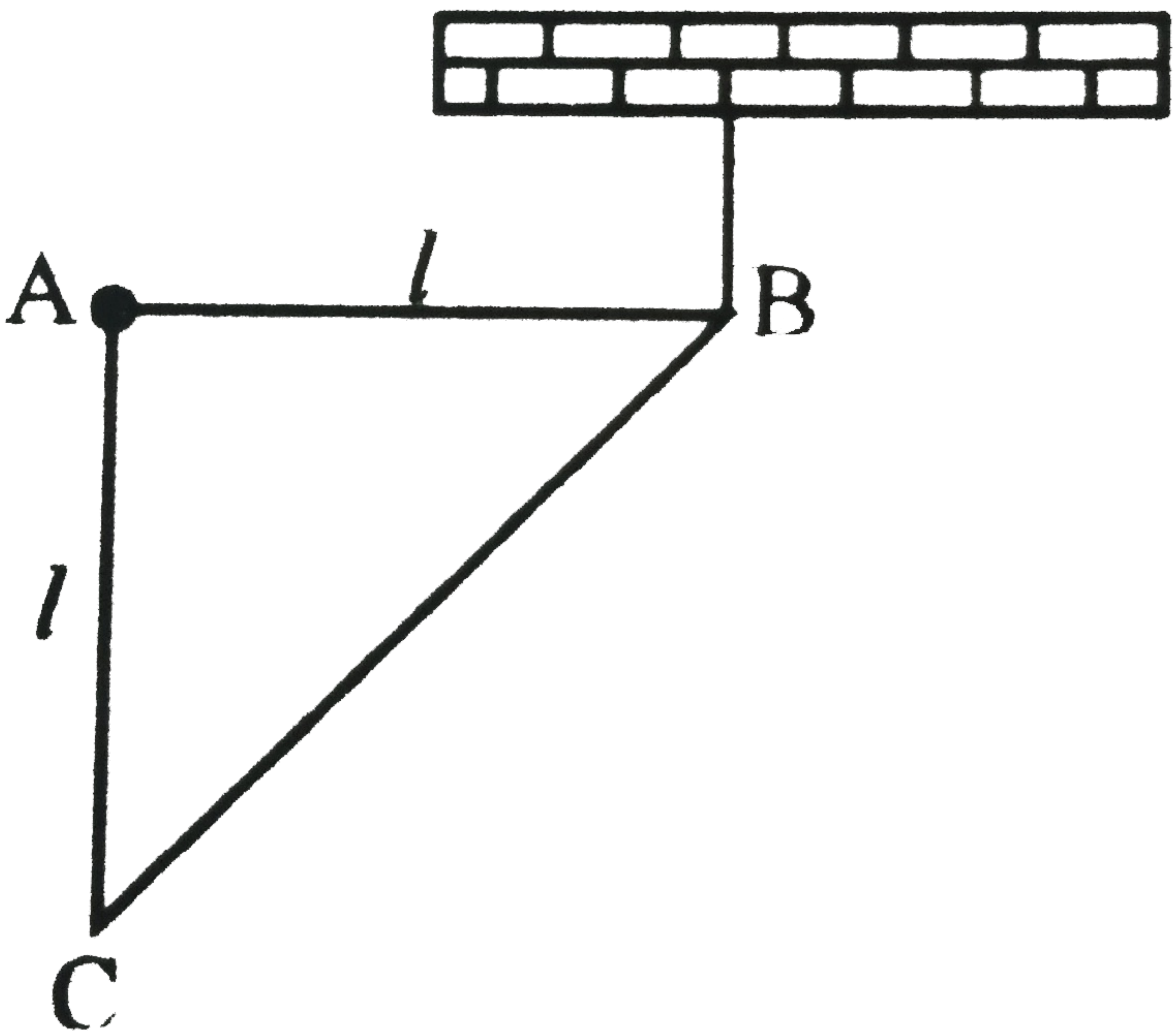
$$I = \frac{mL^2}{12} + mR^2$$



& K.E. of rod w.r.t disc $= \frac{1}{2} m\omega^2 \left[R^2 + \frac{L^2}{12}\right]$

Q-37 - 12229759

A right triangular plate ABC of mas m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A is :



(A) $\frac{mg}{3}$

(B) $\frac{2mg}{3}$

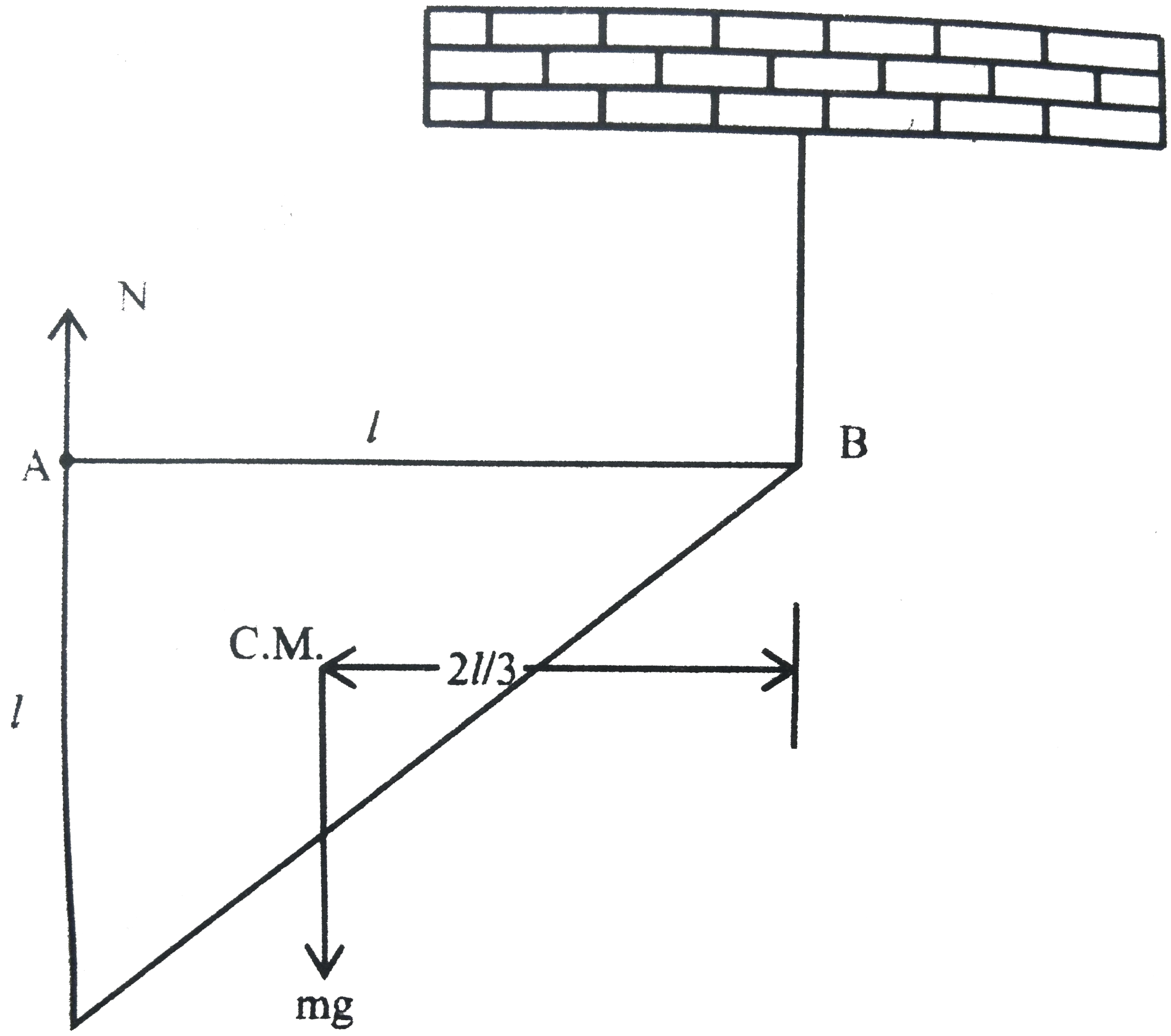
(C) $\frac{mg}{2}$

(D) mg

CORRECT ANSWER: B

SOLUTION:

Taking moment of force about B. $(CW \oplus)$



$$Nl - mg \times \frac{2l}{3} = 0, N$$

$$= \frac{2mg}{3}$$

.

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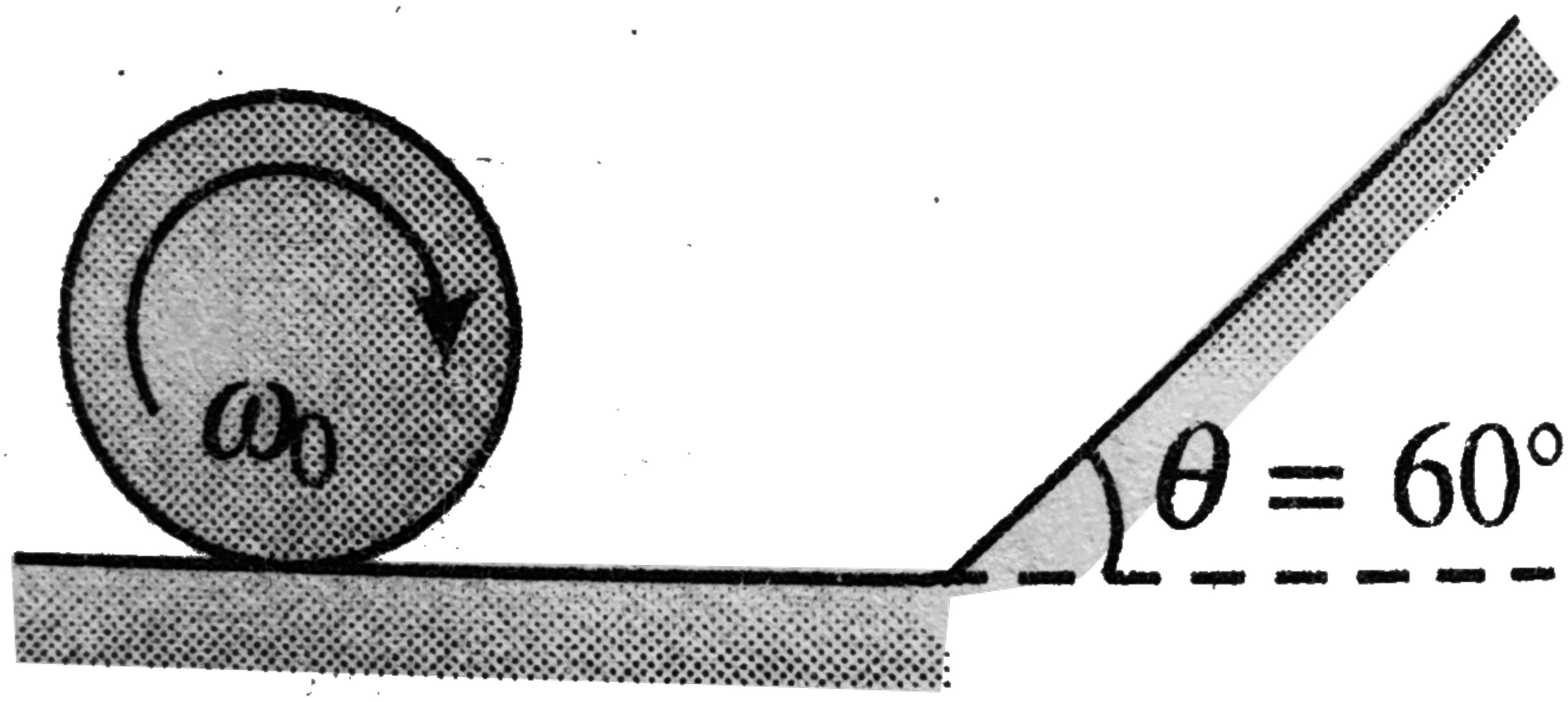
Q-38 - 11301488

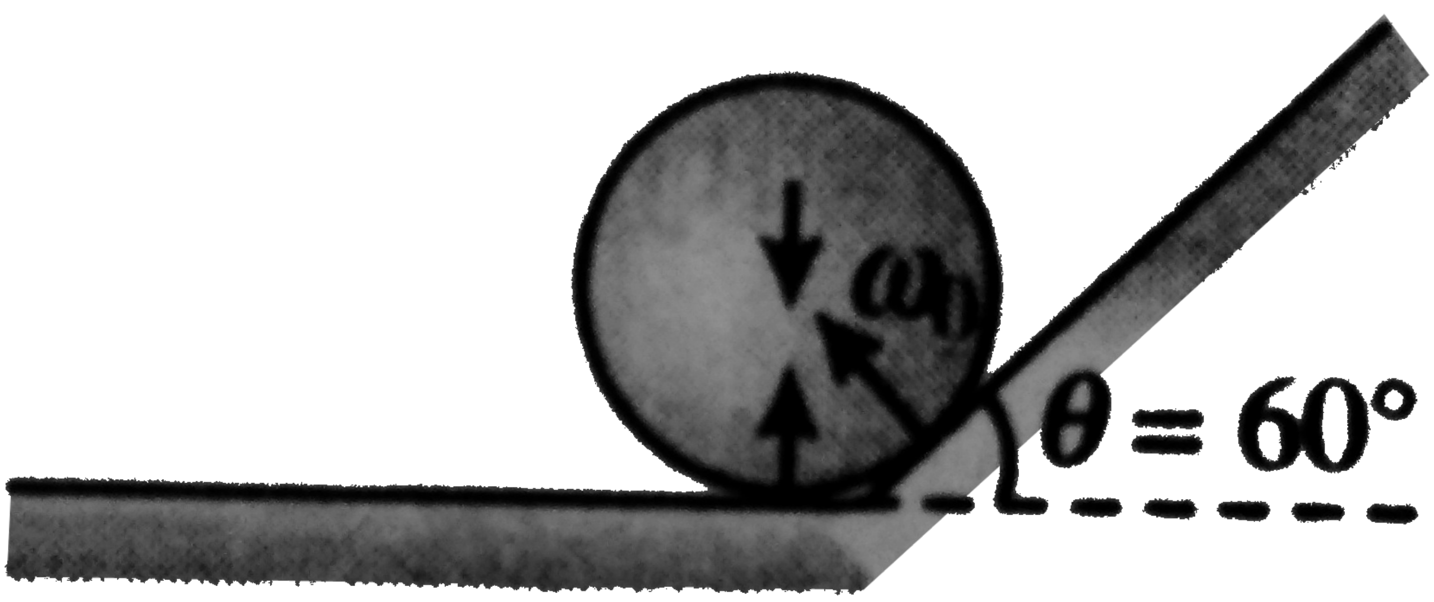
A uniform solid sphere of radius R, rolling without sliding on a horizontal surface with an angular velocity ω_0, meets a rough inclined plane of inclination $\theta = 60$. The sphere starts pure rolling up the plane with an angular velocity ω Find the value of ω.



CORRECT ANSWER: $\frac{9}{14} OMEGA_0$

SOLUTION:

Using the conservation of angular momentum about the point of contact with the inclined plane.



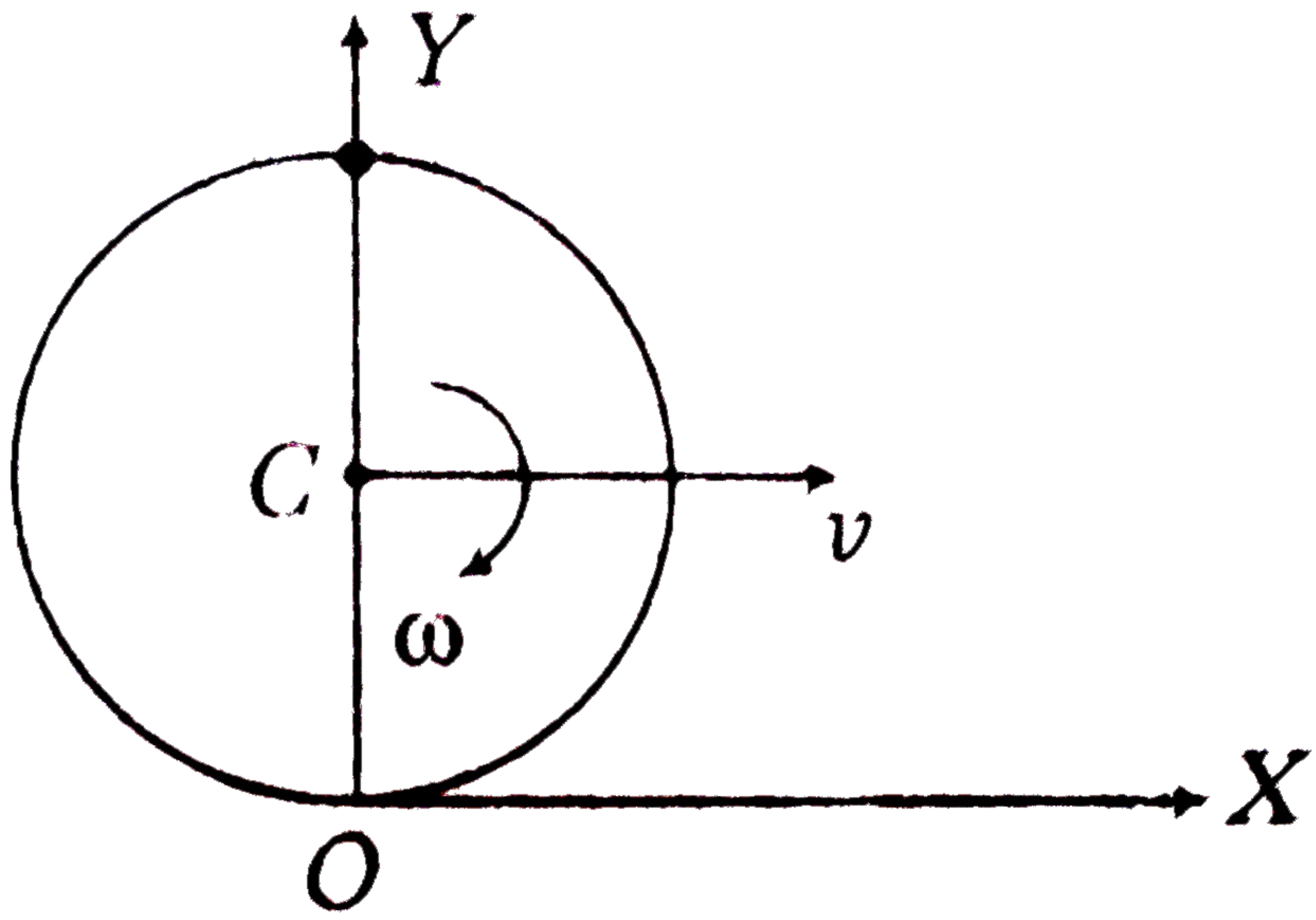
$$\frac{2}{5}mR^2\omega_0$$
$$+ mv_0 R\cos 60^2$$
$$= \frac{7}{5}mR^2\omega$$

$$\frac{2}{5}mR^2\omega_0 + \frac{m\omega_0 R^2}{2}$$
$$= \frac{7}{5}mR^2\omega$$

$$\frac{9}{10}mR^2\omega_0 = \frac{7}{5}mR^2\omega$$
$$\therefore \omega = \frac{9}{14}\omega_0$$

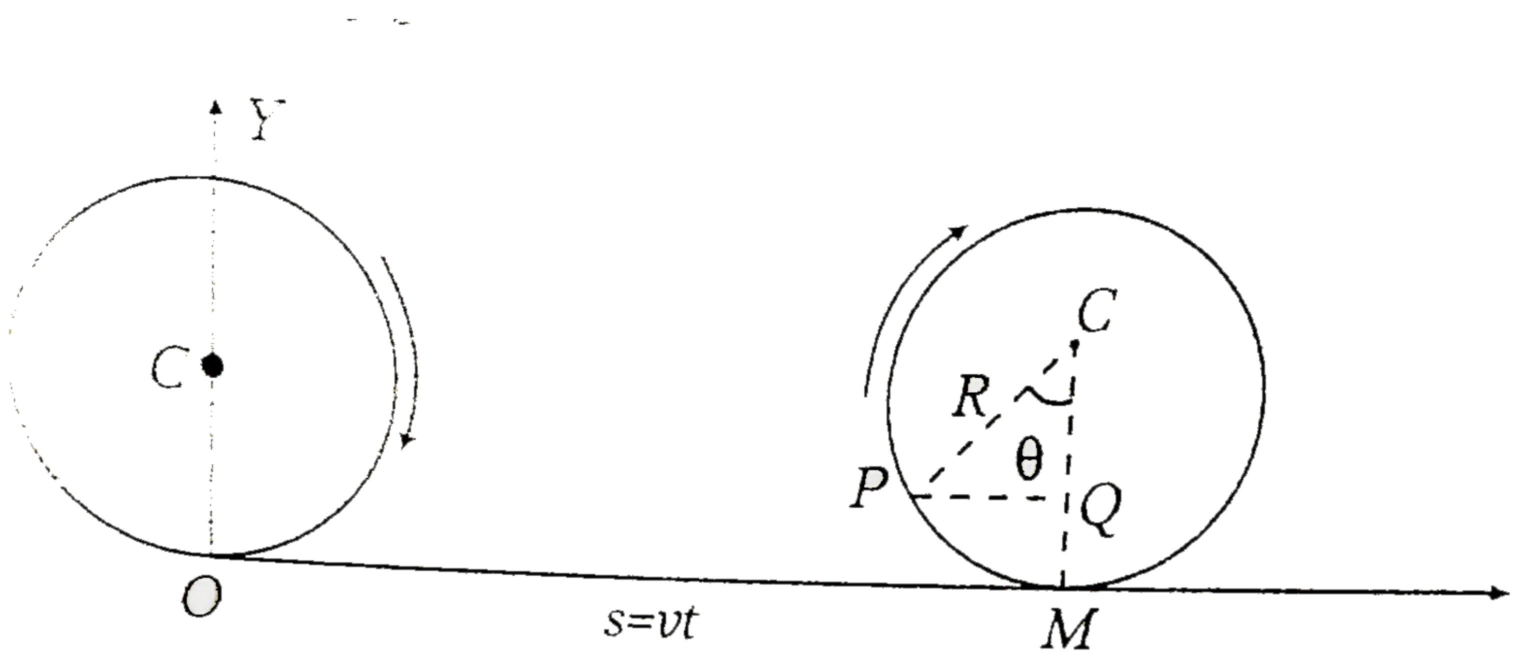
Q-39 - 18707614

A disc of radius R start at time $t = 0$ moving along the positive X-axis with linear speed b and angular speed ω. Find the x and y-coordinates of the bottommost point at any time t.



SOLUTION:

At time t, the bottommost point will rotate an angle $\theta = \omega t$ with respet to the centre of the disc c. The centre C will travel a distance $s = vt$



In the figure ,

$$PQ = R\sin\theta$$
$$= R\sin\omega t$$

and

$$CQ = R\cos\theta$$
$$= R\cos\omega t$$

Coordinates of points P and at time t are,

$$x = OM - PQ = vt$$
$$- R\sin\omega t$$

and

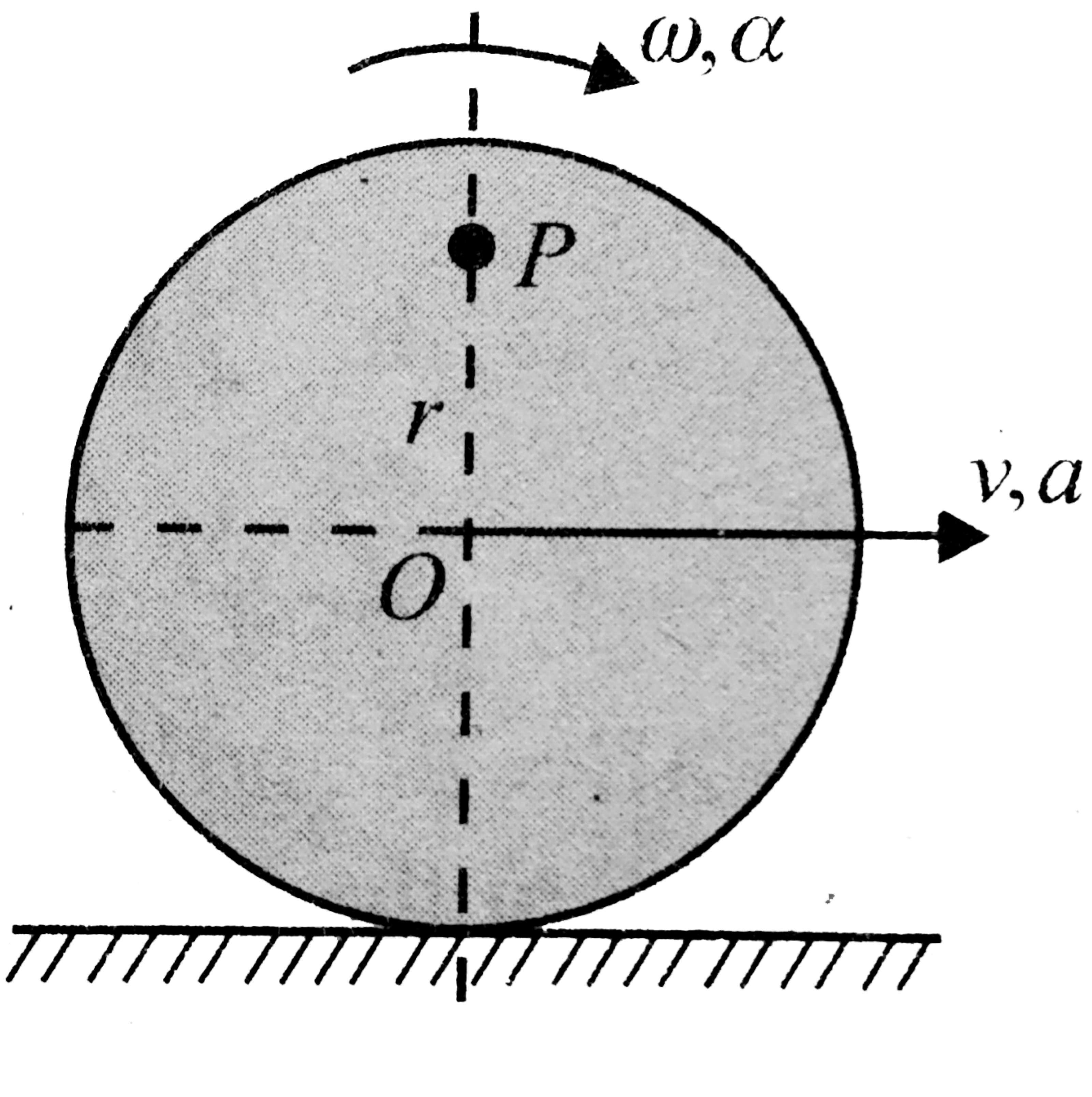
$$y = CM - CQ = R$$
$$- R\cos\omega t$$

$$\therefore (x, y)$$
$$\equiv (vt - R\sin\omega t, R$$
$$- R\cos\omega t)$$

Q-40 - 11301185

A disc of radius R rolls on a horizontal ground with linear acceleration a and angular acceleration α as shown in Fig. The magnitude of acceleration of point P as shown in the figure at an instant when its linear velocity is v and angular velocity is ω will be

a



(A) $\sqrt{(a+r\alpha)^2+(r\omega^2)^2}$

(B) $\frac{ar}{R}$

(C) $\sqrt{r^2\alpha^2 + r^2\omega^4}$

(D) $r\alpha$

CORRECT ANSWER: A

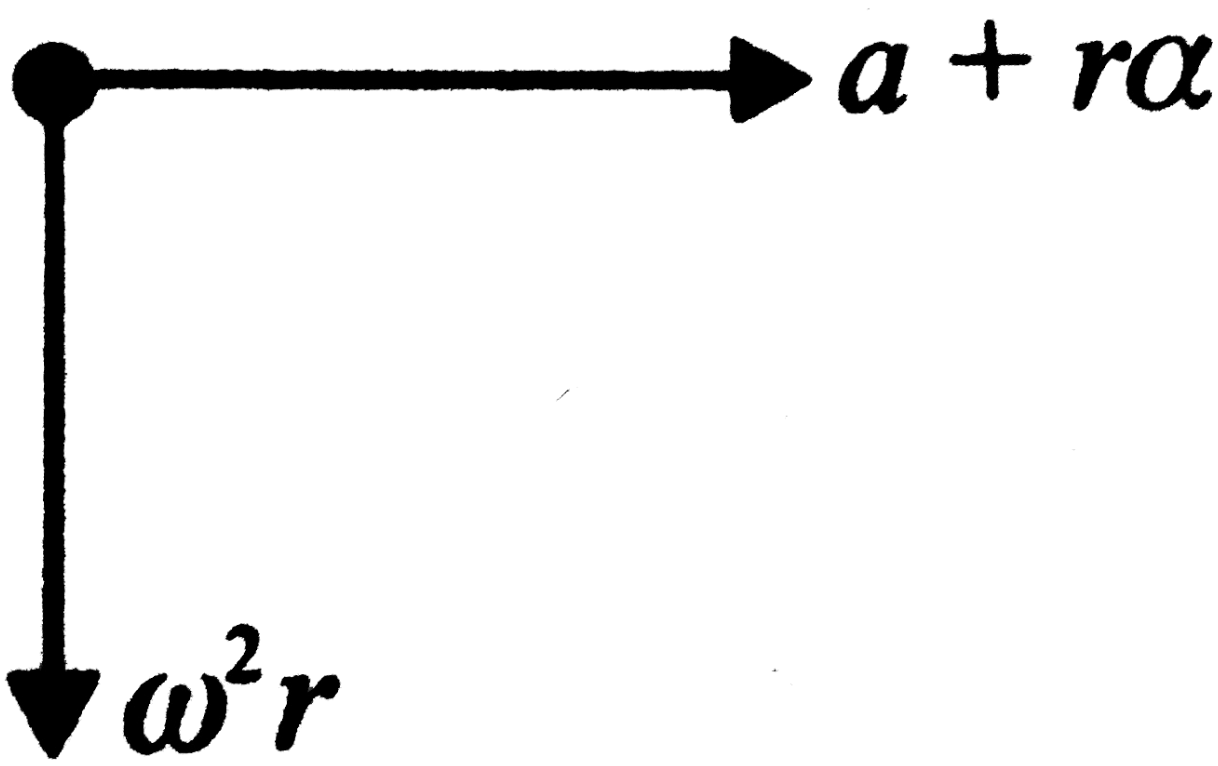
SOLUTION:

FBD of point P

In the earth frame, we have

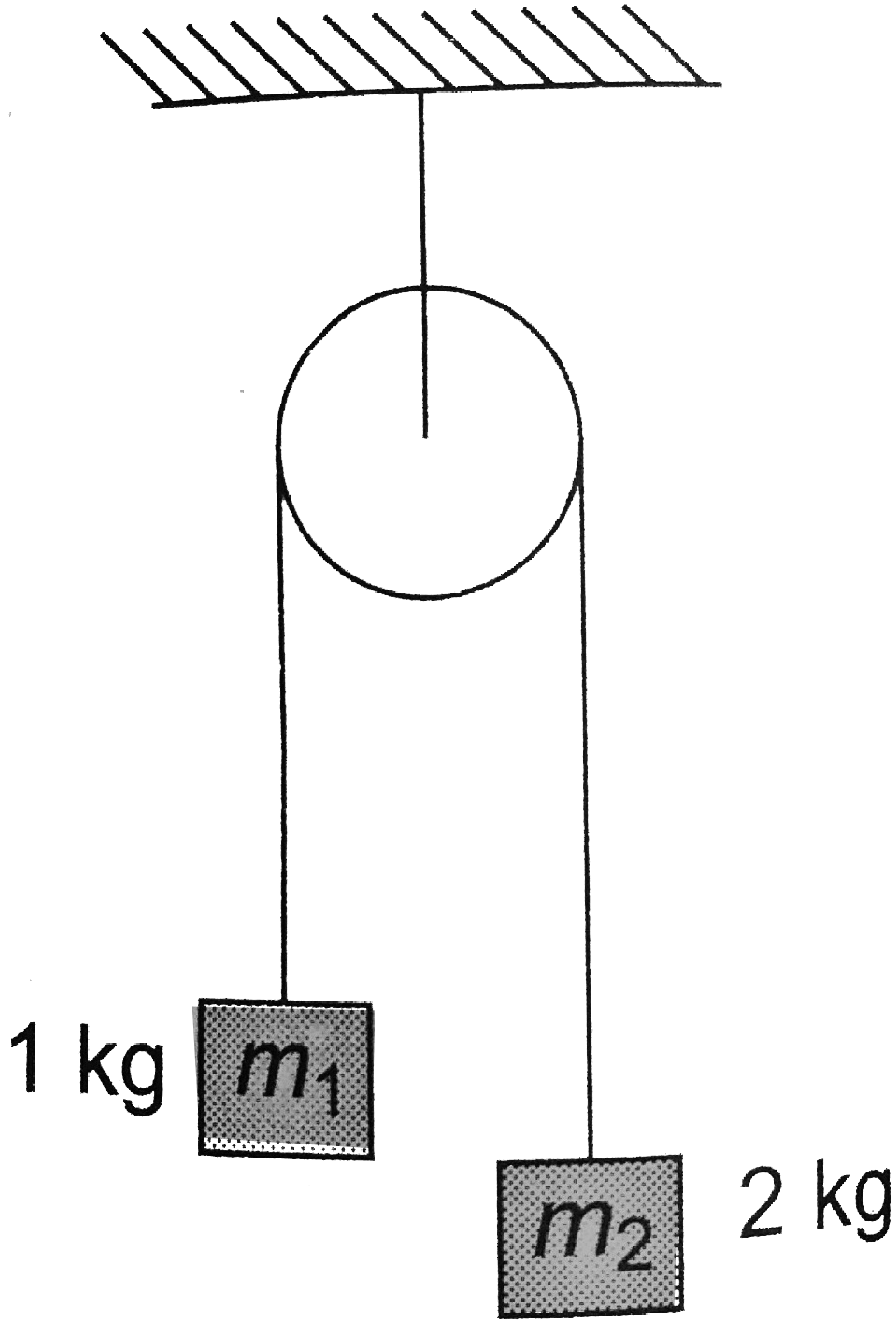
So, resultant

$$= \sqrt{(a + r\alpha)^2 + (\omega^2 r)^2}$$



Q-41 - 11748002

Two masses $m_1 = 1kg$ and $m_2 = 2kg$ are connected by a light inextensible string and suspended by means of a weightness pulley as shown in the figure. Assuming that both the masses start from rest, the distance travelled by the centre of mass in two seconds is $(Take g = 10ms^{-2}$.



(A) $\frac{20}{9}m$

(B) $\frac{40}{9}m$

(C) $\frac{2}{3}m$

(D) $\frac{1}{3}m$

CORRECT ANSWER: A

SOLUTION:

(a) Here, $m_1 = 1kg, m_2 = 2kg$

The acceleration of the system is

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$= \frac{(2 - 1)g}{(1 + 2)} = \frac{g}{3}$$

$$= \frac{10}{3}$$

Acceleration of the centre of mass is

$$a_{cm} = \frac{m_1 + a_1 a_2}{m_1 + m_2}$$

$$= \frac{1(- a) + 2(a)}{1 + 2}$$

=

$$\frac{\left(\frac{-g}{3}\right)+2\left(\frac{g}{3}\right)}{3}=\frac{g}{9}$$

$$=\frac{10}{9}$$

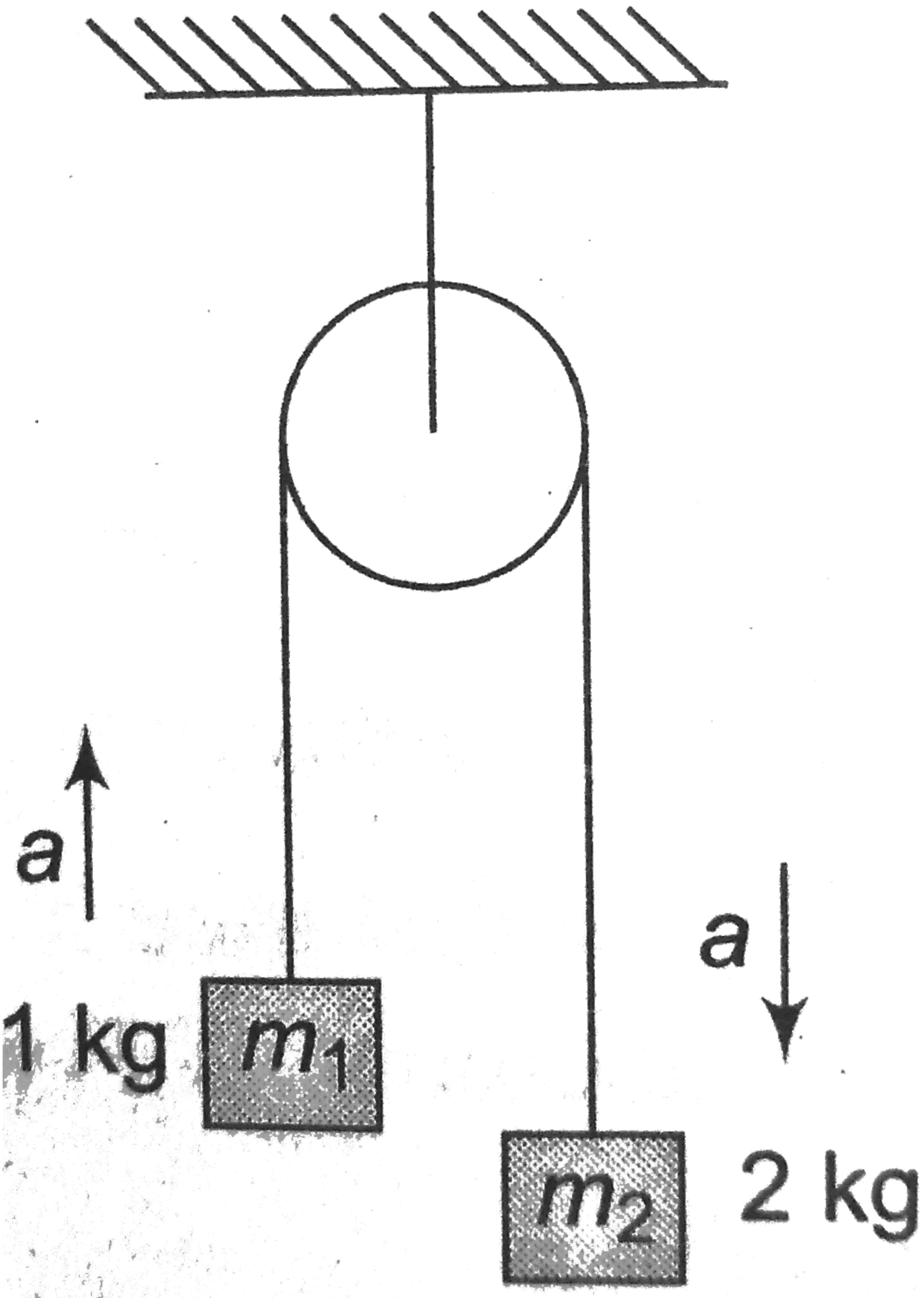
The distance travelled by the centre of mass in two

seconds is

$$S=\frac{1}{2}a_{cm}t^{2}=\frac{1}{2}$$

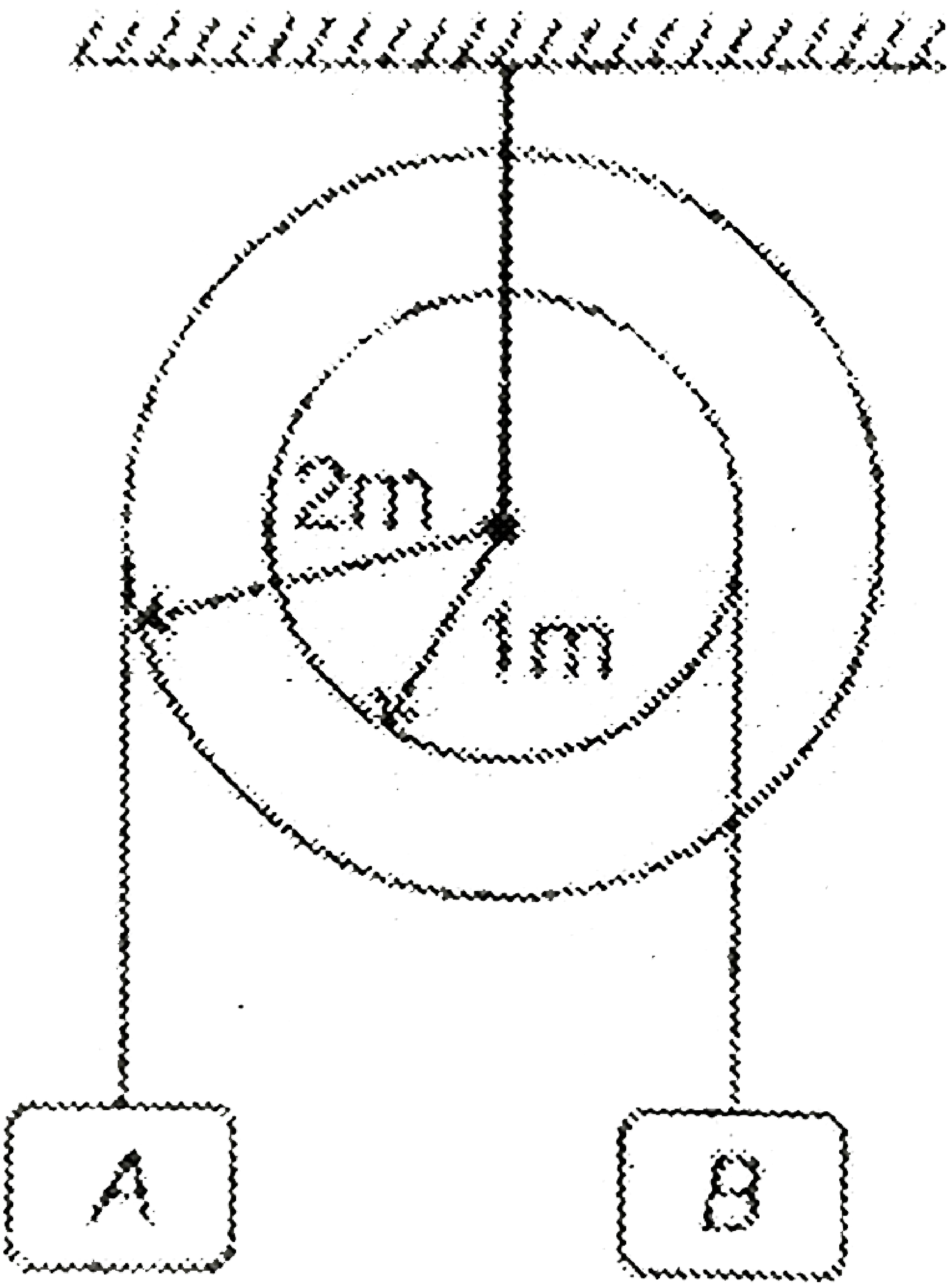
$$\times\frac{10}{9}\times(2)^{2}=\frac{20}{9}m$$

.



Q-42 - 17459057

In the pully system shown, if radii of the bigger and smaller pulley are 2m and 1m respectively and the acceleration of block A is $5m/s^2$ in the downward direction, then the acceleration of block B will be



(A) $0m/s^2$

(B) $5m / s^2$

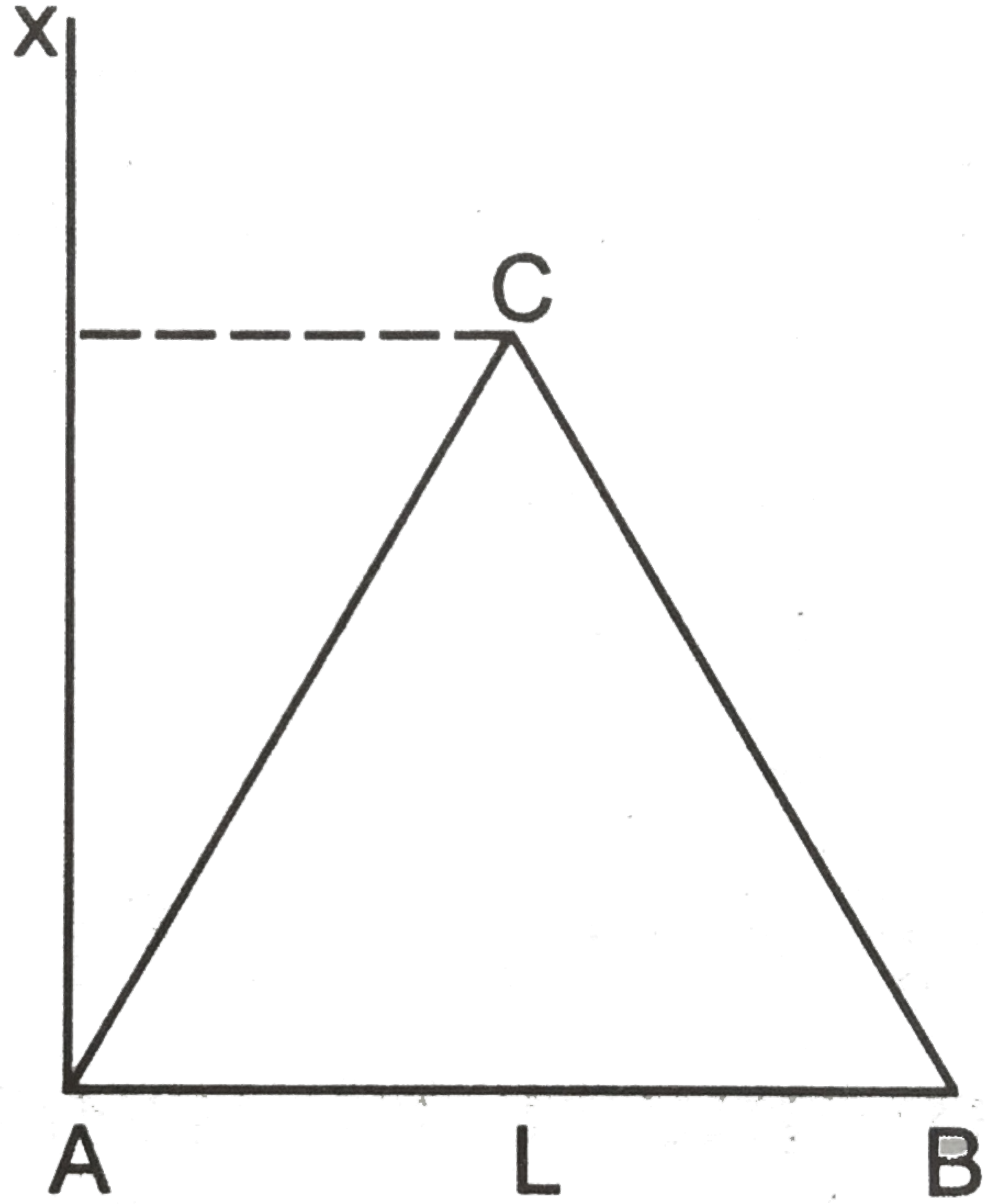
(C) $10m / s^2$

(D) $\frac{5}{2}m / s^2$

CORRECT ANSWER: D

Q-43 - 9519568

Three particles, each of mass m are situated at the vertices of an equilateral triangle ABC of side L figure. Find thee moment of inertia of the system about the line AX perpendicular to AB in the plane of ABC



SOLUTION:

Perpendicular distance of A from AX=0

Perpendicular distance B from AX=L

Perpendicular distance of DC from AX=L/2

Thus, the moment of inertia of the particle at A=0, of the particle at $B = mL^2$ and of the particle at

$C = m\left(\frac{L}{2}\right)^2$. The moment of inertia of the three particle system about AX is

$$0 + mL^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{5mL^2}{4}$$

Note that the particle on the axis do not contribute to the moment of inertia.

Q-44 - 13025952

Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along the collinear axes passing through their center of mass and perpendicular to their plane. If the two are made to rotate combindly along the same axis the rotational $K.E.$ of system will be

(A) $\dfrac{I_1\omega_1 + I_2\omega_2}{2(I_1 + I_2)}$

(B) $\dfrac{(I_1 + I_2)(\omega_1 + \omega_2)^2}{2}$

(C) $\dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$

(D) None of these

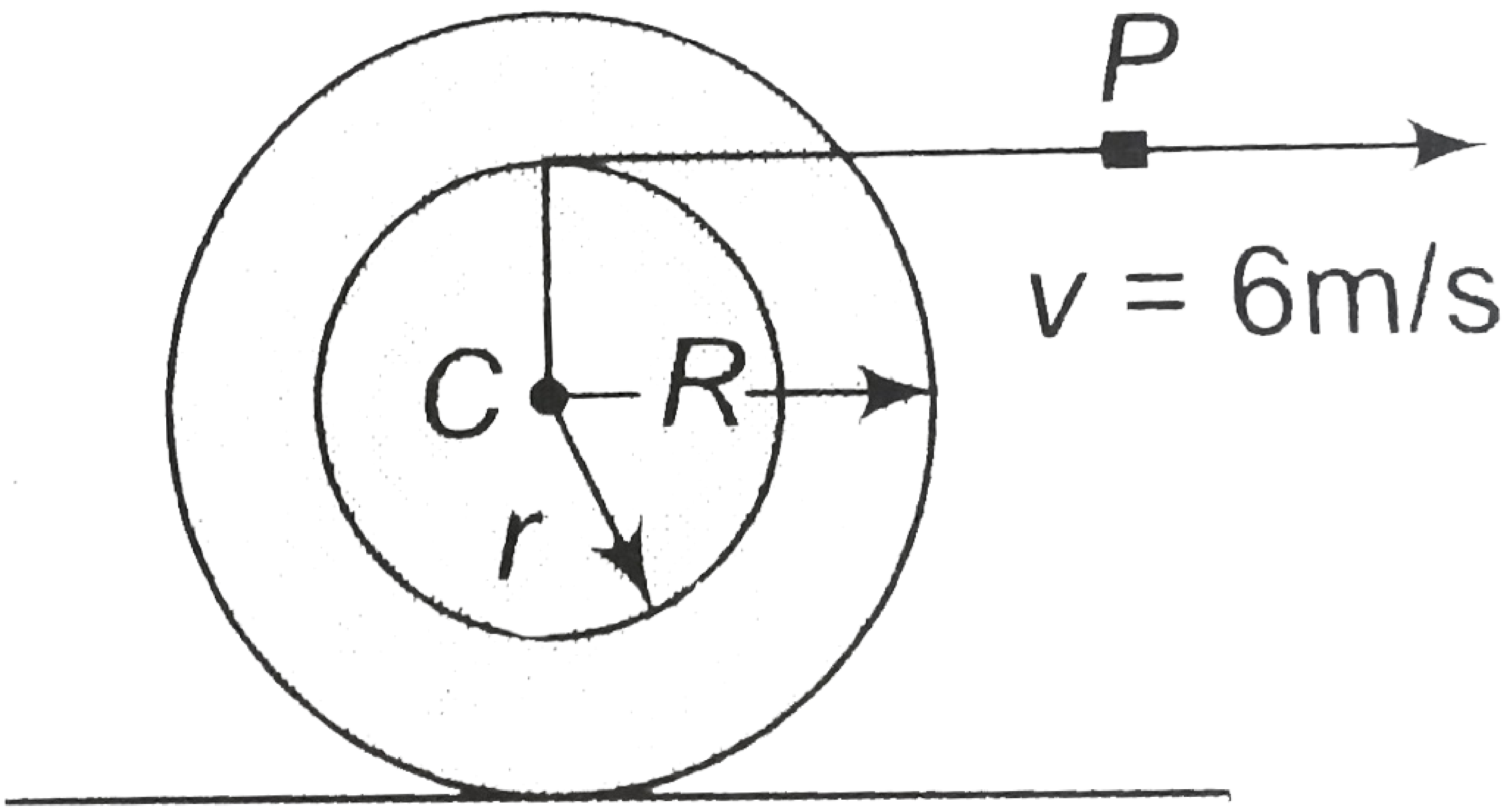
SOLUTION:

$$(I_1\omega_1 + I_2\omega_2)$$
$$= (I_1 + I_2)\omega'$$

$$\omega' = \frac{I_1\omega_1 + I_2\omega_2}{(I_1 + I_2)}$$
$$K_f = \frac{1}{2}(I_1 + I_2)\omega'^2$$
$$= \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$$

Q-45 - 11748191

A bobbin is pushed along on a rough stationary horizontal surface as shown in the figure. The board is kept horizontal and there is no slipping at any contact points. The distance movedby the board when distance moved by the axis of the bobbin is l is



.

(A) $l\left(1+\frac{r}{2R}\right)$

(B) $l\left(2+\frac{r}{2R}\right)$

(C) $l\left(1+\frac{r}{R}\right)$

(D) $l\left(1+\frac{2r}{R}\right)$

CORRECT ANSWER: C

SOLUTION:

(c) For no slipping , $v_{cm} = \omega R$

$$v_B = \omega r + v_{cm}$$
$$= v_{cm}\left(1 + \frac{r}{R}\right)$$

$$\frac{l_B}{l} = \frac{v_B}{v_{cm}}$$
$$= \left(1 + \frac{r}{R}\right) \Rightarrow l_B$$
$$= l\left(1 + \frac{r}{R}\right)$$

.

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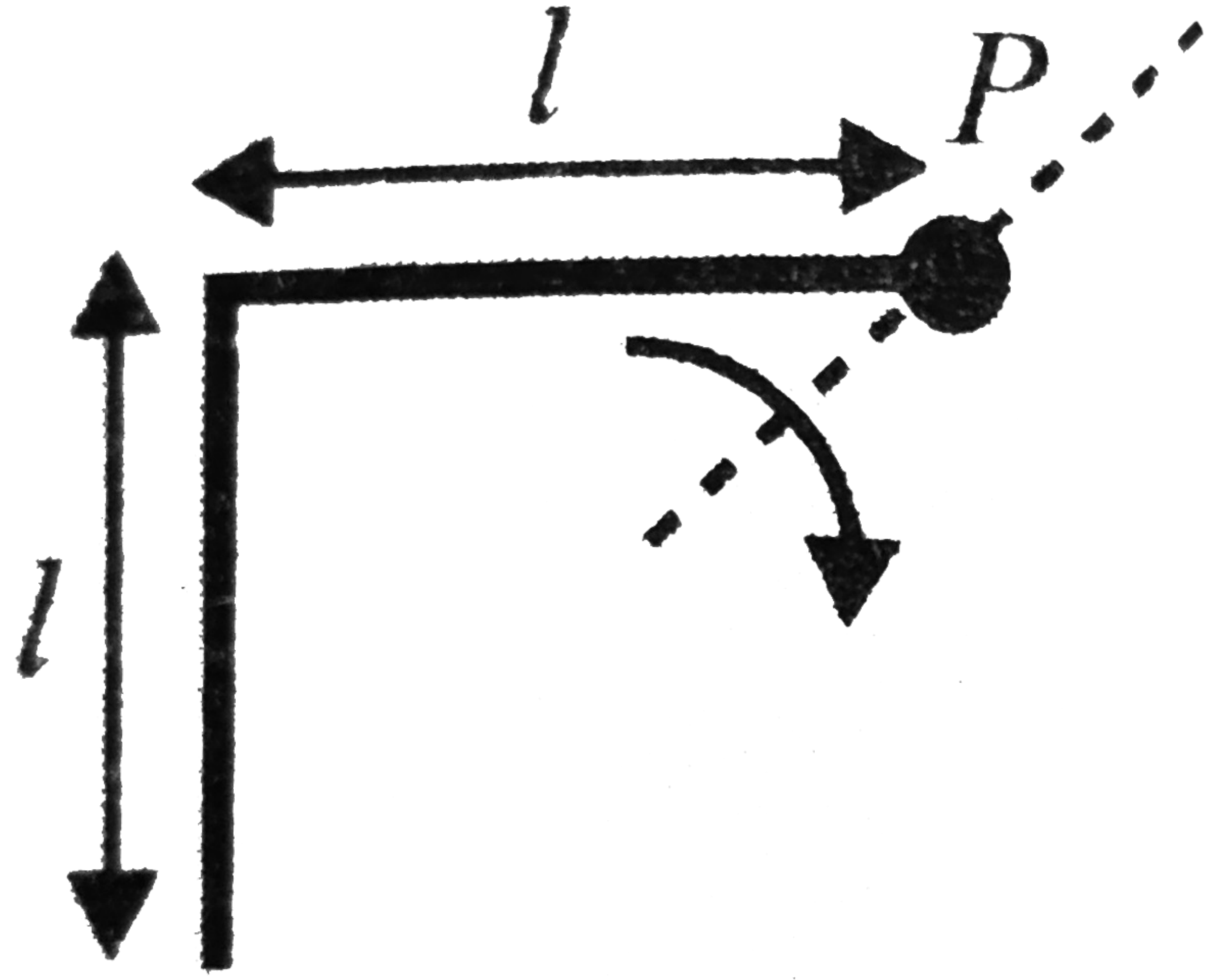
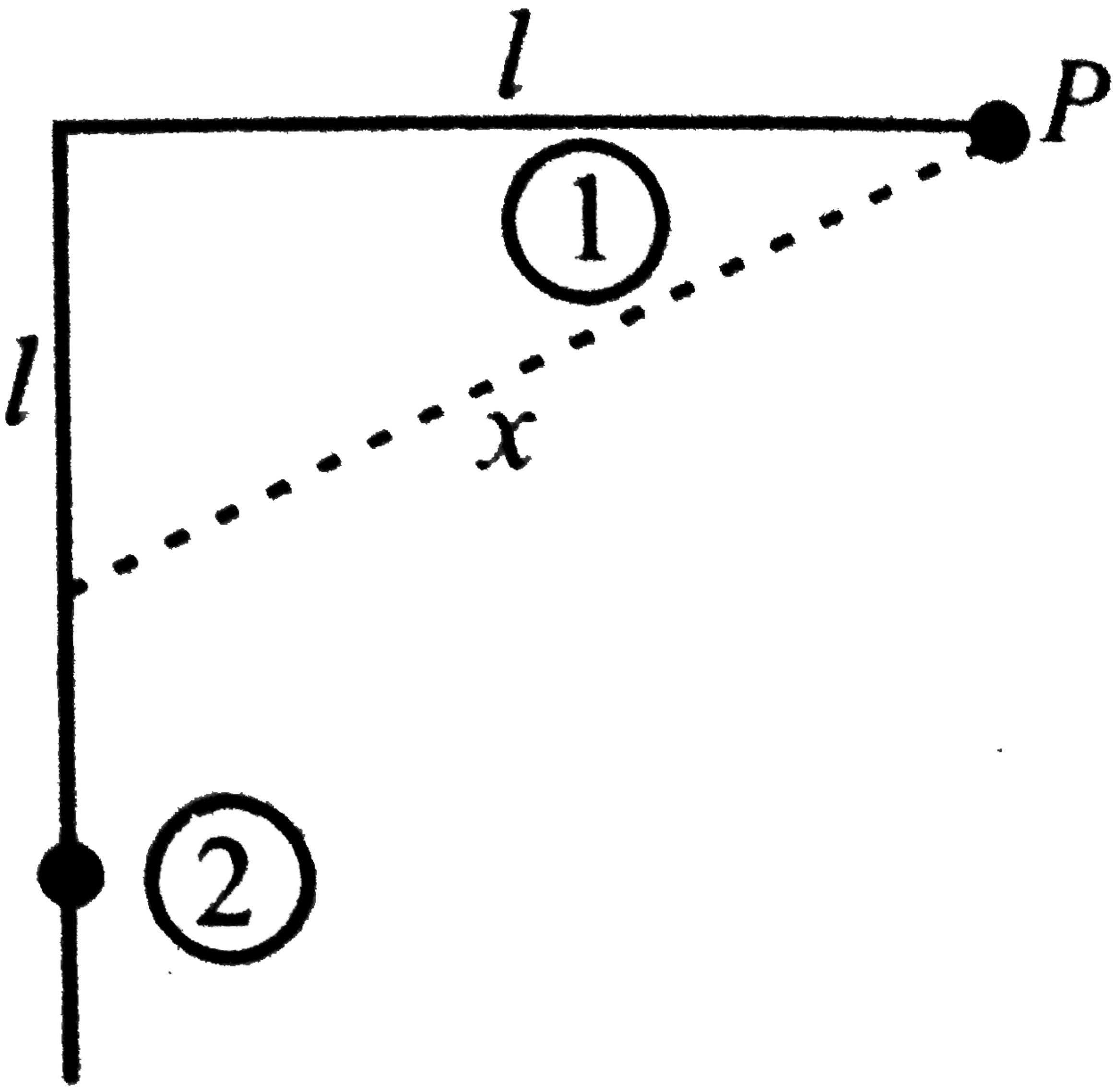
Q-46 - 11301103

Find the moment of inertia of the two uniform joint rods about point P as shown in Fig. Use parallel axis theorem. Mass of each rod is M.



SOLUTION:

$$I_P = I_{P_1} + I_{P_2}$$



$$I_P = \frac{Ml^2}{3} + \left(\frac{Ml^2}{12} + Mx^2\right)$$

$$x^2 = l^2 + \left(\frac{l}{2}\right)^2 = \frac{5l^2}{4}$$

$$\Rightarrow I_P = \frac{Ml^2}{3}$$

$$+ \left[\frac{Ml^2}{12} + M\left(\frac{5l^2}{4}\right)\right]$$

$$= \frac{5}{3}Ml^2$$

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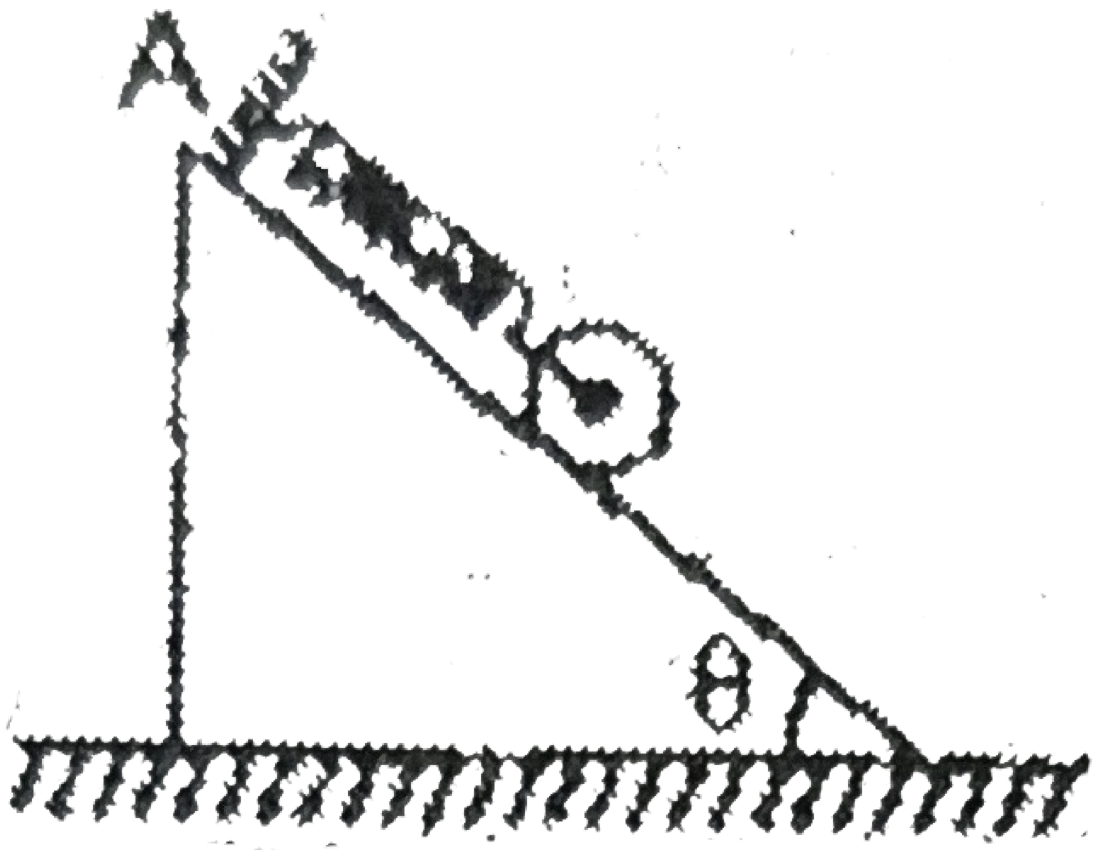
Q-47 - 16687388

A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle 8 with horizontal. The cylinder is connected to a spring of force constant k at the centre, the other side of which is connected to a fixed support at A. The cylinder is released when the spring is unstretched. The force of friction (f):



(A) is always upwards

(B) is always downwards

(C) is initially upwards and then becomes downwards

(D) is initially upwards and then becomes zero

CORRECT ANSWER: C

Q-48 - 18707645

The moment of inertia of a solid cylinder of mass M, length 2R and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of cylinder is I_2

(A) $l_2 - l_1 = MR^2$

(B) $l_2 = l_1$

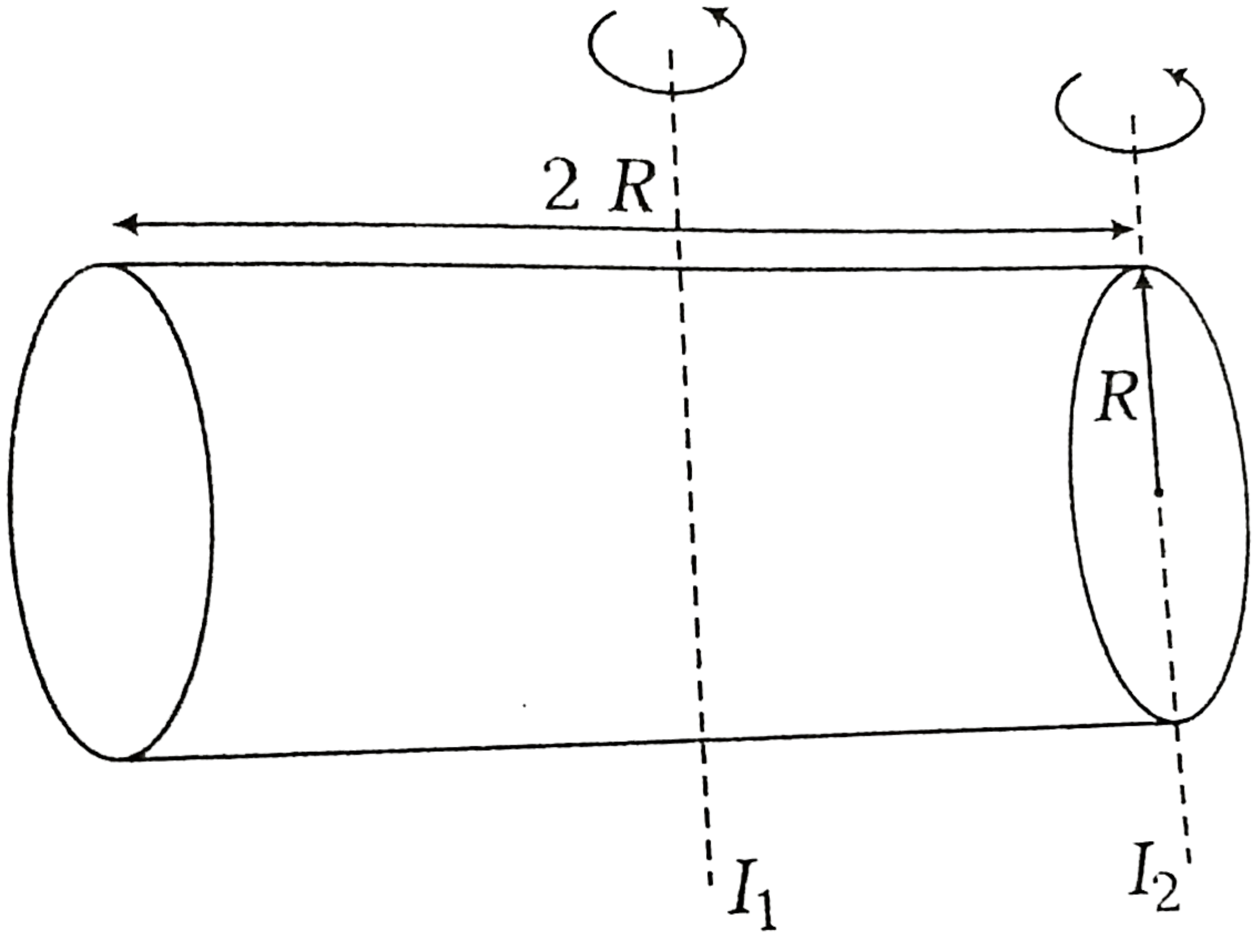
(C) $\frac{l_2}{l_1} = \frac{19}{12}$

(D) $l_1 - l_2 = MR^2$

CORRECT ANSWER: A

SOLUTION:

From theorem of parallel axes, $I_2 = I_1 + MR^2$



Q-49 - 13076655

If I_1 is moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre and I_2 is its moment of inertia when it is bent into a shape of a ring then (Axis passing through its centre and perpendicular to its plane)

(A) $I_2 = \dfrac{I_1}{4\pi^2}$

(B) $I_2 = \dfrac{I_1}{\pi^2}$

(C) $\frac{I_2}{I_1} = \frac{\pi^2}{3}$

(D) $\frac{I_2}{I_1} = \frac{3}{\pi^2}$

CORRECT ANSWER: D

SOLUTION:

$$I_1 = \frac{ML^2}{12}, I_2$$

$$= MR^2, R = \frac{L}{2\pi}$$

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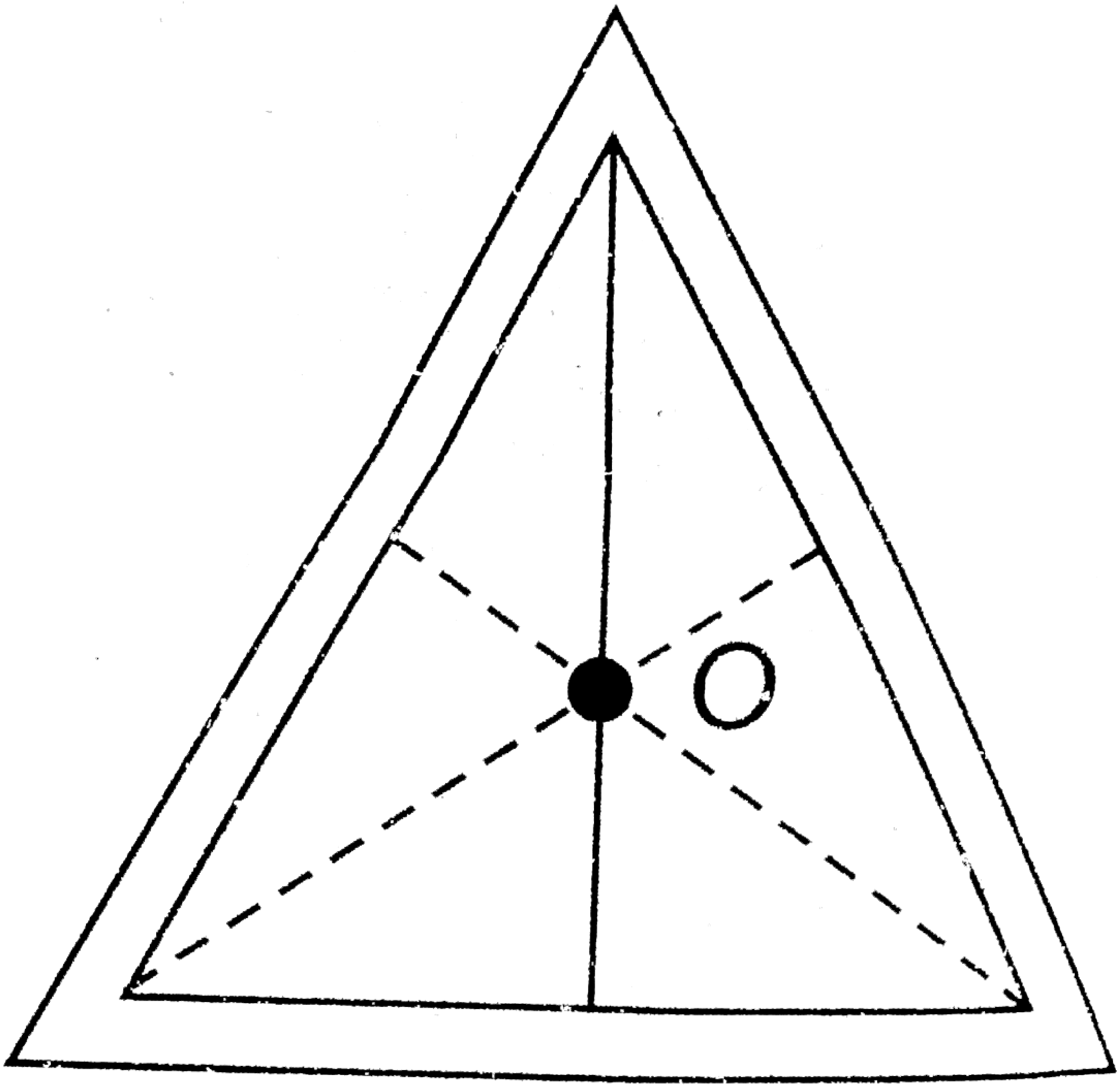
Q-50 - 11748037

A rod of mass Mkg and length Lm is bent in the from of an equilateral tringle as shown in Gig. The moment of inertia of the triangle about a vertical axis perpendicular to the plane of the trinangle and passing through the centre (in units of kgm^2) is.



(A) $\dfrac{ML^2}{12}$

(B) $\dfrac{ML^2}{54}$

(C) $\dfrac{ML^2}{162}$

(D) $\dfrac{ML^2}{108}$

CORRECT ANSWER: B

SOLUTION:

(b) Mass of each side $= \frac{M}{3}$

$x = \frac{1}{3}\left[\frac{L}{3}\cos 30^{\square}\right]$

$= \frac{L/3}{2\sqrt{3}}$

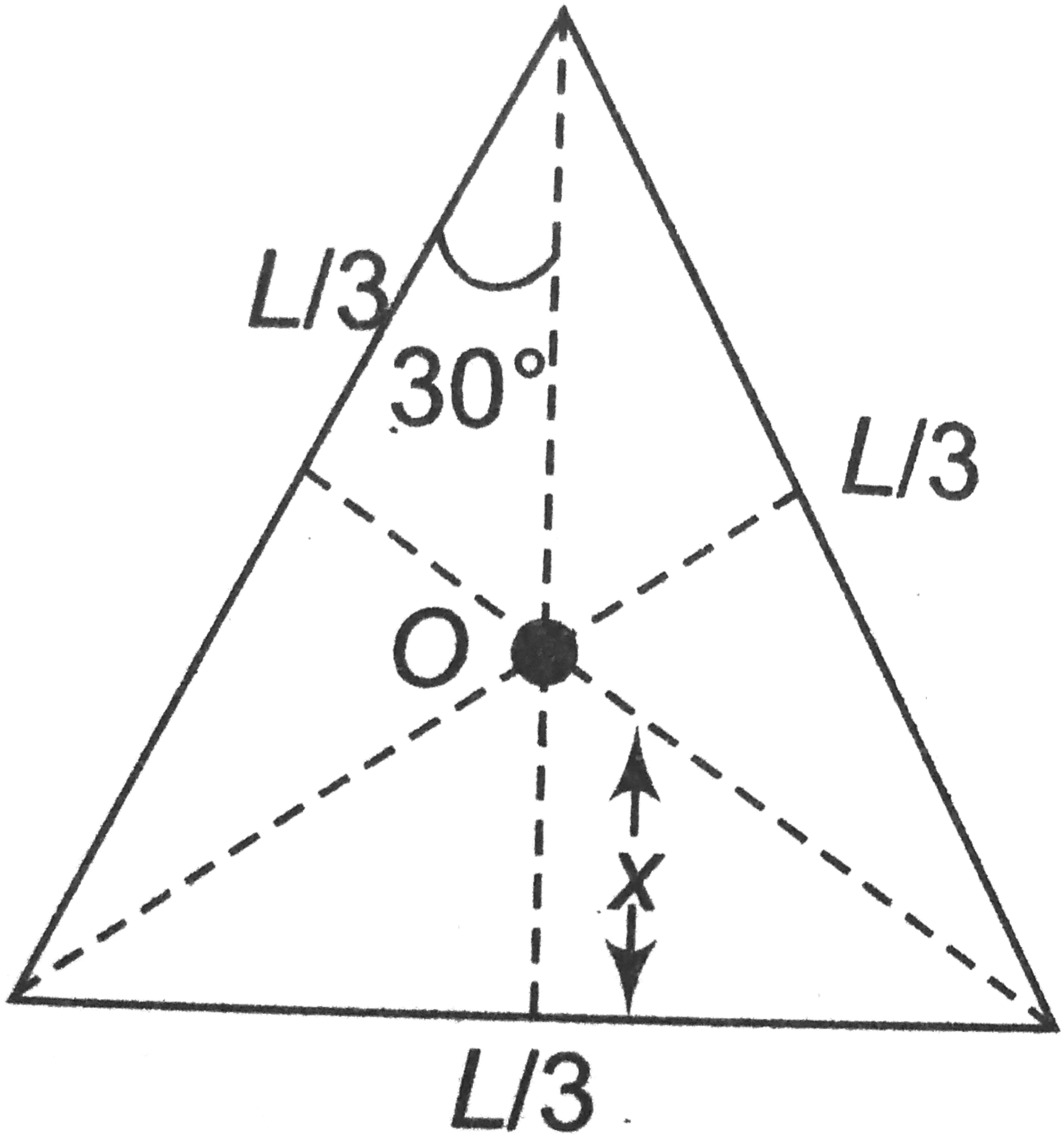
moment of interia of one side about O :

$I = \left[\frac{1}{12}\frac{M}{3}\left(\frac{L}{3}\right)^2\right.$

$\left. + \frac{M}{3}x^2\right] = \frac{ML^2}{162}$

moment of interia of whole triangle about

$O: 3I = \frac{ML^2}{54}$.



Q-51 - 11301166

Two rings of same radius and mass are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to the plane of one of the rings is (mass the ring $= m$, radius $= r$)

(A) $\frac{1}{2} m r^2$

(B) $m r^2$

(C) $\frac{3}{2} m r^2$

(D) $2 m r^2$

CORRECT ANSWER: C

SOLUTION:

Because the plane of two ringns are mutually perpendicular and centres are coincident, hence n axis, which is passing through the centre of one of the rings and to its plane, will be along the diameter of other ring.

Hence, moment of inertia of the system

$$= I_{CM} + I_{\text{diameter}}$$

$$= m r^2 + \frac{m r^2}{2}$$

$$= \frac{3}{2} m r^2$$

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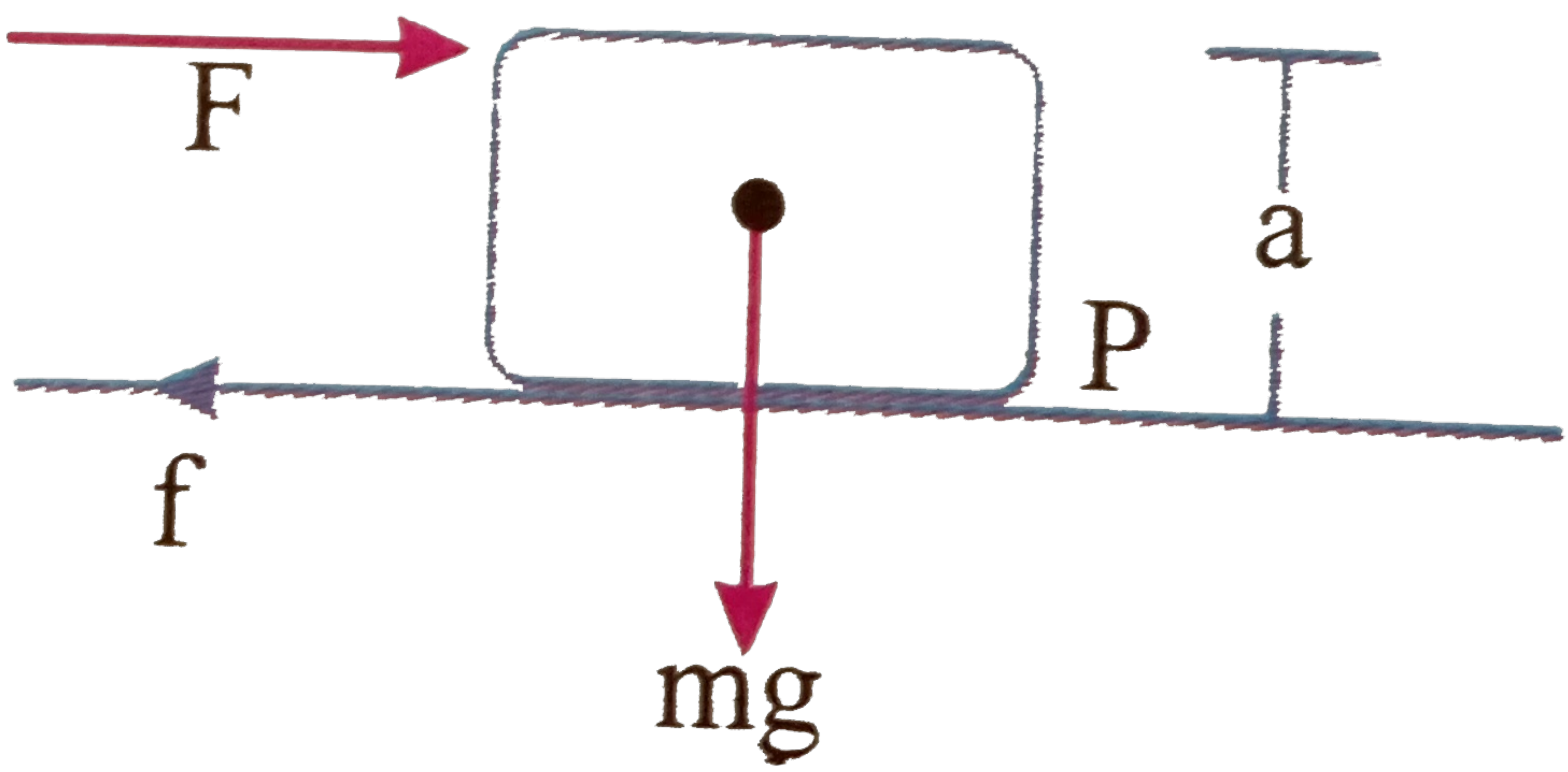
Q-52 - 13076192

A force F is applied on the top of a cube as shown in the figure. The coefficient of friction between the cube and the ground is μ. If F is gradually increased, find the value of μ for which the cube will topple before sliding.



SOLUTION:

Let m be the mass of the cube and 'a' be the side of the side. The cube will slide if $F > \mu mg......(1)$ and it will

topple if torque of F about P is gtreater than torque of

mg about P i.e., $Fa > \left(\frac{a}{2}\right)$ mg or $F > \frac{1}{2}mg......(2)$

From equations (1) and (2) we see that cube will topple

before sliding if $\mu > \frac{1}{2}$.

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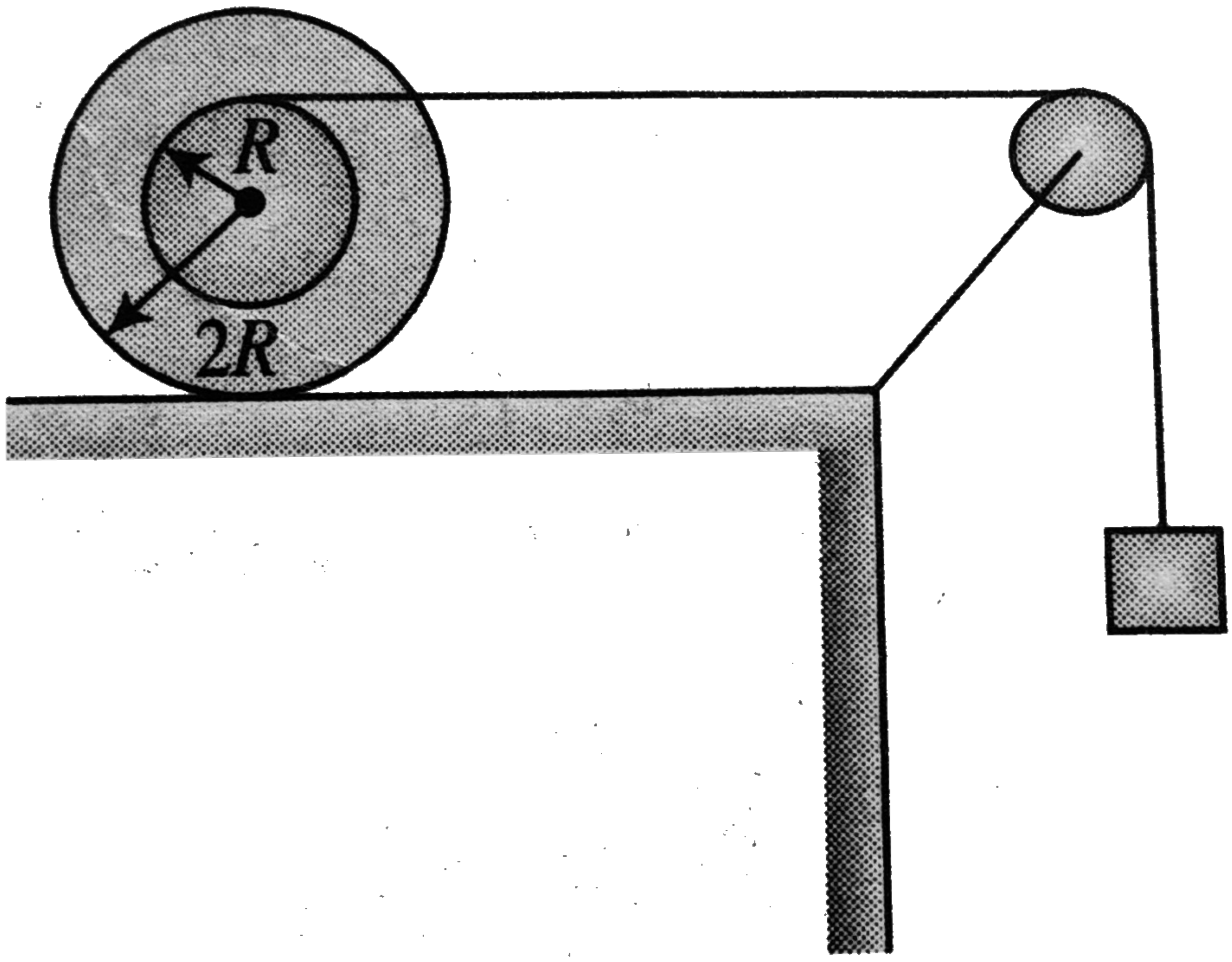
Q-53 - 11301615

In the figure shown mass of both, the spherical body and block is m. Moment of inertia of the spherical body about centre of mass is $2mR^2$. The spherical body rolls on the horizontal surface. There is no slipping at any surfaces in contact. The ratio of kinetic energy of the spherical body to that of block is



(A) $3 / 4$

(B) $1 / 3$

(C) $2 / 3$

(D) $1 / 2$

CORRECT ANSWER: C

SOLUTION:

Let v be the linear velocity of centre of mass of the sphereical body and ω its angular velocity about centre of mass. Then $\omega = \frac{v}{2R}$

KE of spherical body

$$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$K_1 = \frac{1}{2}mv^2$$

$$+ \frac{1}{2}(2mR)^2\left(\frac{v^2}{4R^2}\right)$$

$$= \frac{3}{4}mv^2 \text{.....i}$$

Speed of the block will be

$$v' = (\omega)(3R) = 3R\omega$$

$$= (3R)\left(\frac{v}{2R}\right) = \frac{3}{2}v$$

$\therefore KE$ of block

$$K_2 = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(\frac{3}{2}v\right)^2$$
$$= \frac{9}{8}mv^2$$
......ii

From eqn i and ii $\frac{K_1}{K_2} = \frac{2}{3}$

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Q-54 - 14928243

The moment of inertia of a uniform semicircular wire of mass m and radius r, about an axis passing through its centre of mass and perpendicular to its plane is $mr^2\left(-\frac{k}{\pi^2}\right)$. Find the value of k.

(A) 2

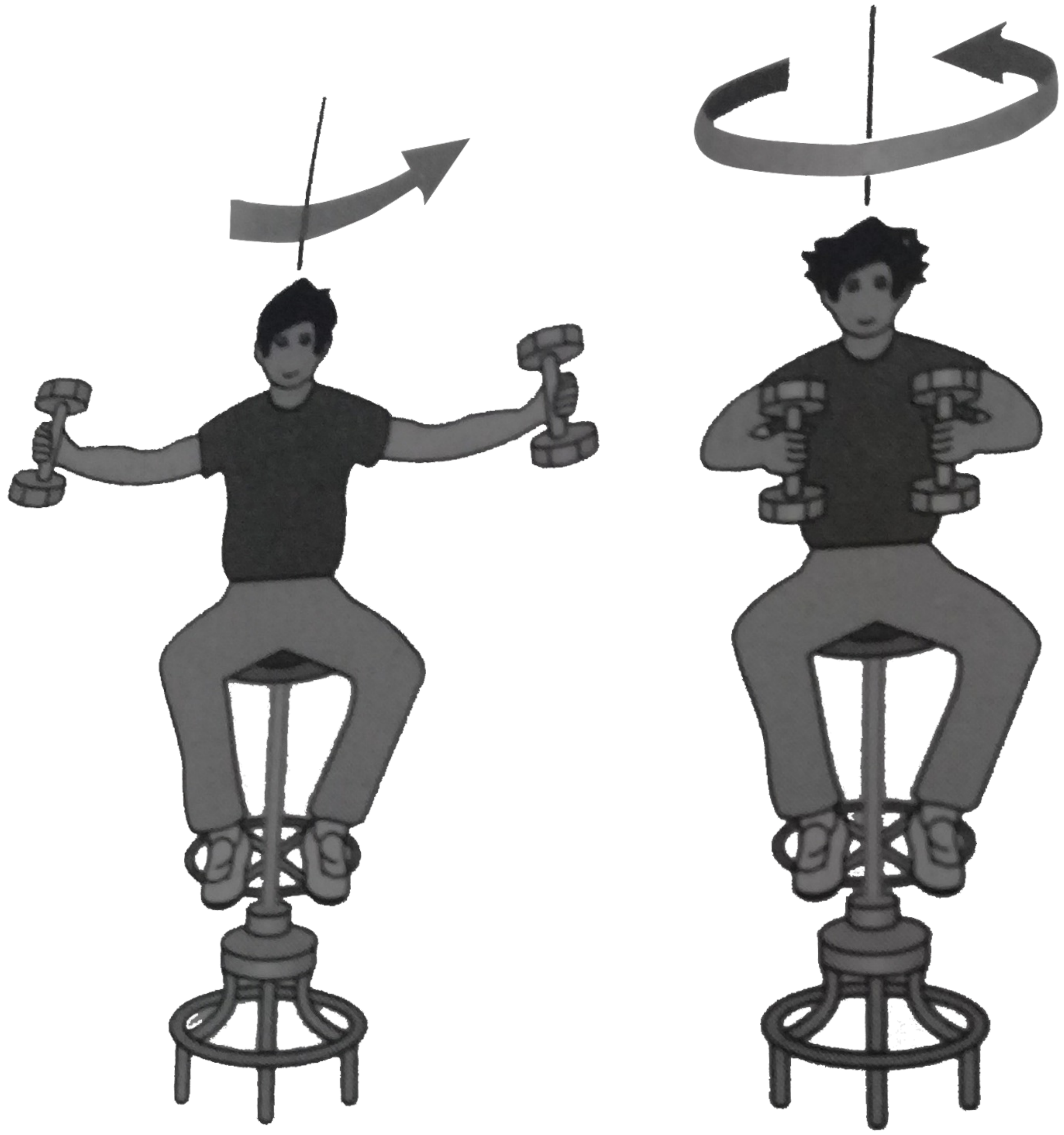
(B) 3

(C) 4

(D) 5

Q-55 - 11748152

A student sits on a freely rotating stool holding dumbbells, each of mass $5.0kg$ (Fig). When his arms are extended horizontally (Fig a), the dumbbells are $1.0m$ from the axis of rotation and the student rotate with an angular speed of $1.0rad / s$. The moment of inertia of the student plus stool is $5.0kg. m^2$ and is assumed to be constant. The student pulls the dumbbells inward horizontally to a position $0.50m$ from the rotationa are (Fig.) The new angular speed of the student is.



(A) $1.5 rad / s$

(B) $2.5 rad / s$

(C) $2.0 rad / s$

(D) $1.25 rad / s$

CORRECT ANSWER: C

SOLUTION:

(c) The total angular momentum of the system of the student, the stool, and the weight about the axis of rotation is given by

$I_{\to tal} = I_{weights}$
$+ I_{student} = 2\left(mr^2\right)$
$+ 5.0kg.m^2$

Before : $r = 1.0m$

Thus,

$I_i = 2(5.0kg)(1.0m)^2$
$+ 5.0kg.m^2 = 15kg$
$.m^2$

After : $r = 0.50m$

Thus,

$I_i = 2(5.0kg)(0.50m)^2$
$+ 5.0kg.m^2 = 7.50kg$
$.m^2$

We now use conservation of angular momentum

$$I_f \omega_f = I_i \omega_i$$

or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$$
$$= \left(\frac{15.0}{7.5}\right)(1.0 rad / s)$$
$$= 2 rad / s$$

.

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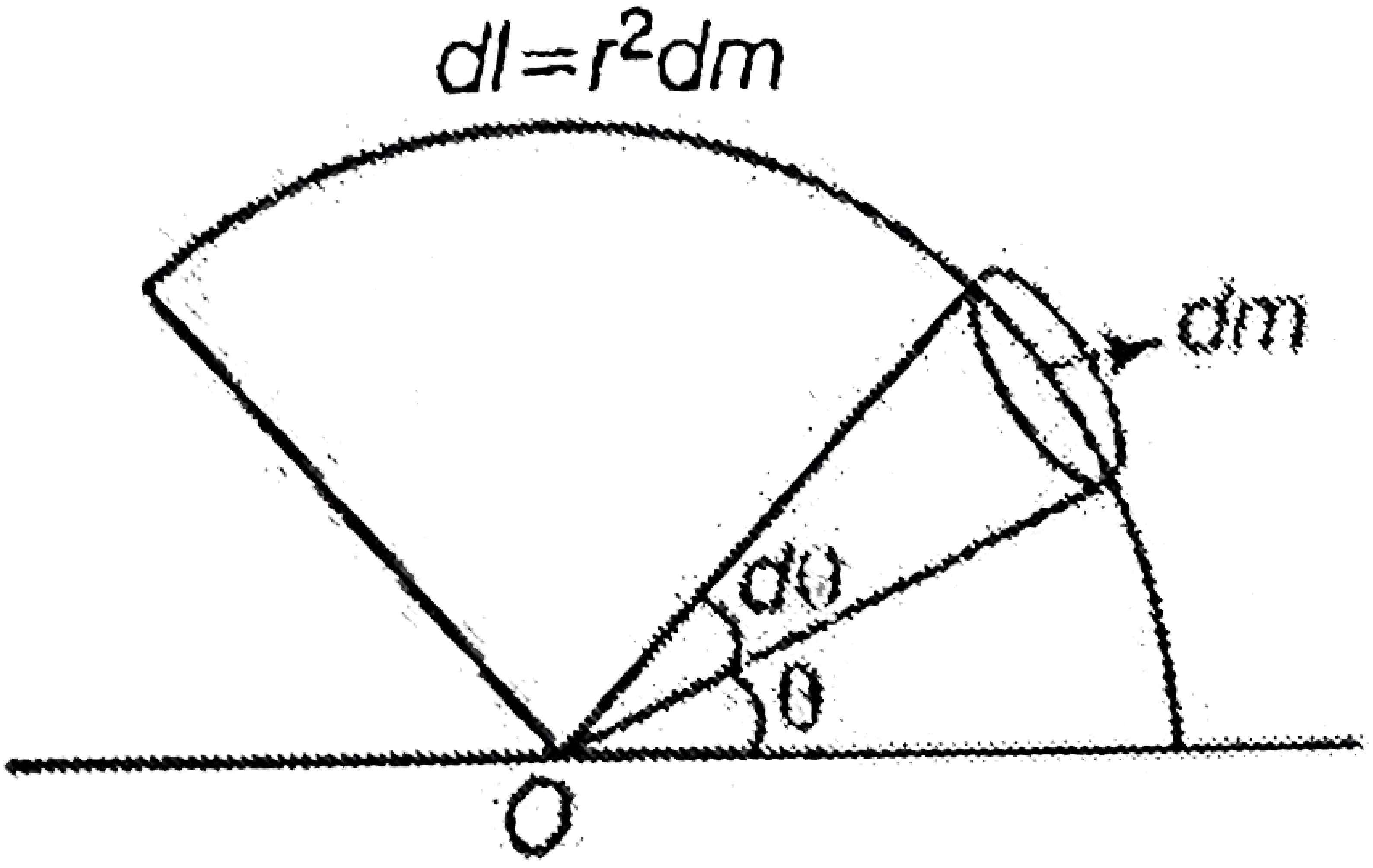
Q-56 - 18253965

An arm making an angle of 120 at the center of ring of mass m and radius r is cut from the ring. The arc is made to rotate about z-axis perpendicular to its plane and passing through the center of the ring. The moment of inertia of the arc about the z-axis is



(A) mr^2

(B) $\frac{mr^2}{3}$

(C) $\frac{mr^2}{2}$

(D) $\left(m\frac{r^2}{4}\right.$

SOLUTION:

b) The mass of ring of per unit angle is $\lambda = \frac{m}{2\pi}$

$\therefore$ dm-= (m)/(2pi) dthetatherefore

$Themomentof$

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r^(2)dmtherefore

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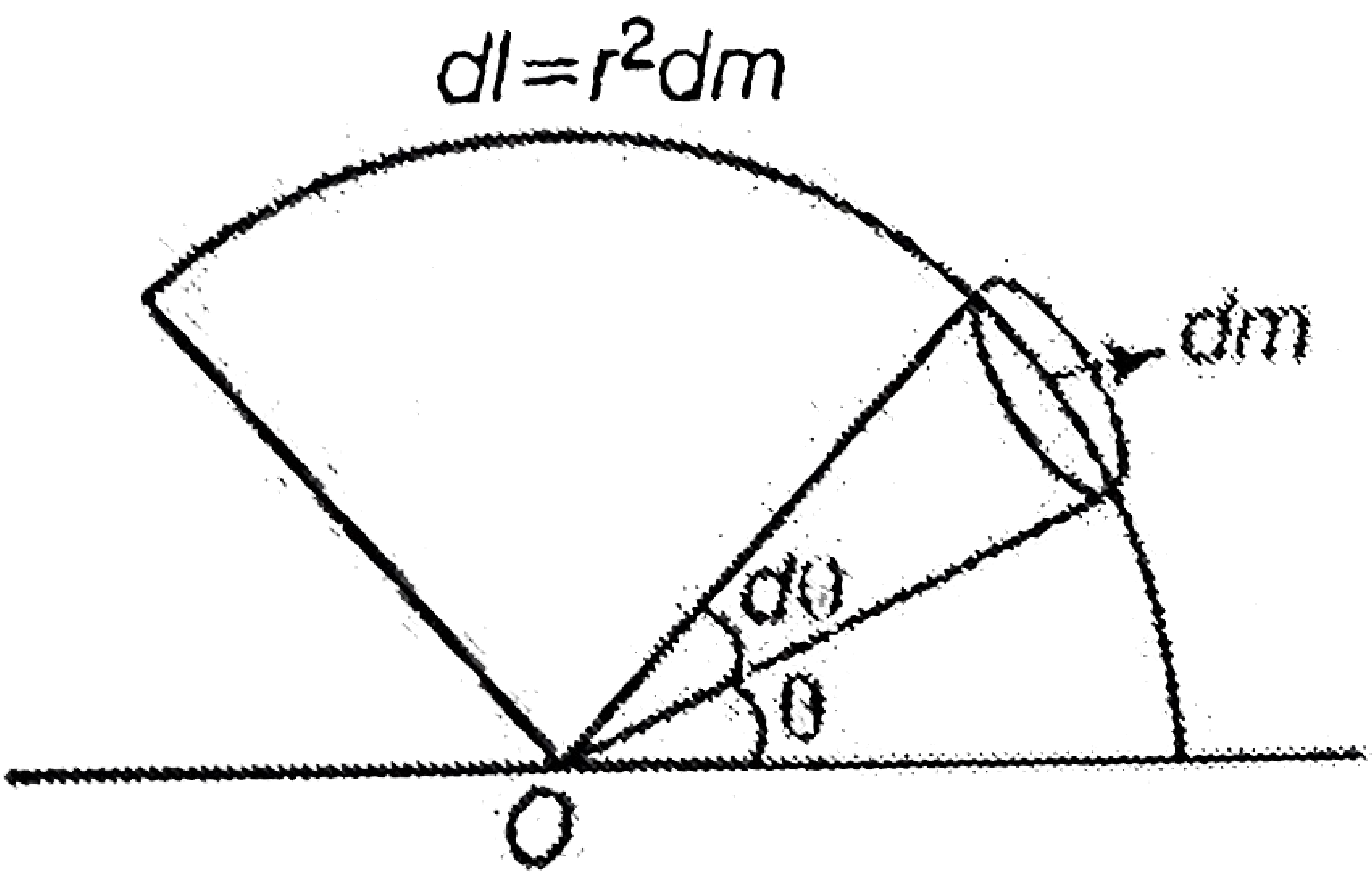
I = r^(2) = (mr^(2)/3 int_(theta=0)^(theta=(2pi//3)))

$dm =$ (mr^(2)/3)`



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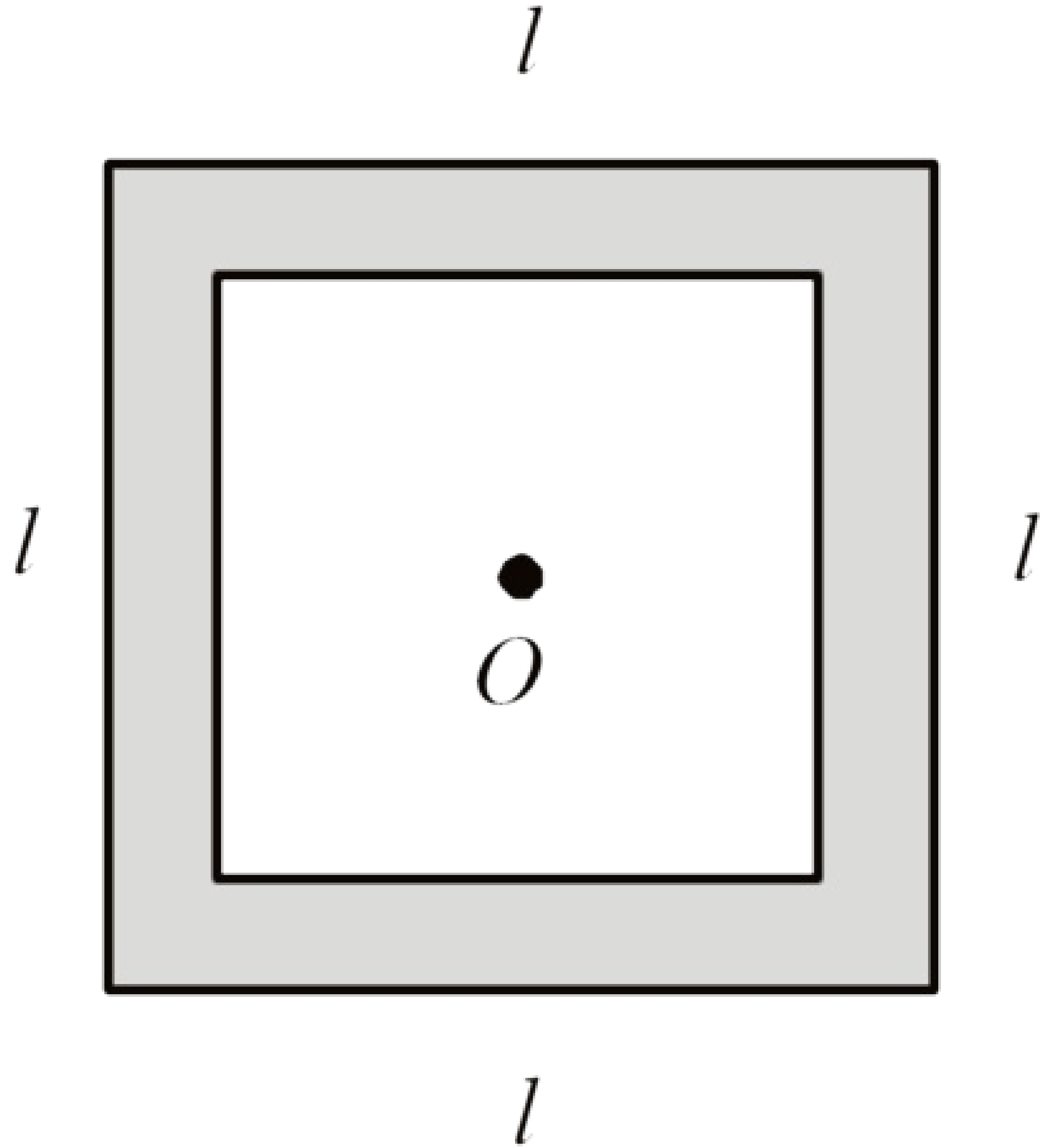
Q-57 - 14798383

Four thin rods of same mass M and same length l, form a square as shown in figure. Moment of inertia of this system about an axis through centre O and perpendicular to its plane is



(A) $\frac{4}{3}Ml^2$

(B) $\frac{Ml^2}{3}$

(C) $\frac{Ml^2}{6}$

(D) $\frac{2}{3}Ml^2$

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Q-58 - 18253948

The tricycle weighing 20 kg has a small wheel symmmetrically placed 1m behind the two large wheels, which are also 1m apart. If the center of gravity of machine is at a horizontal distance of 25 cm behind the front wheels and the rider whose weight is 40 kg is 10 cm behind the front wheels. Then, the thrust on each front wheel is

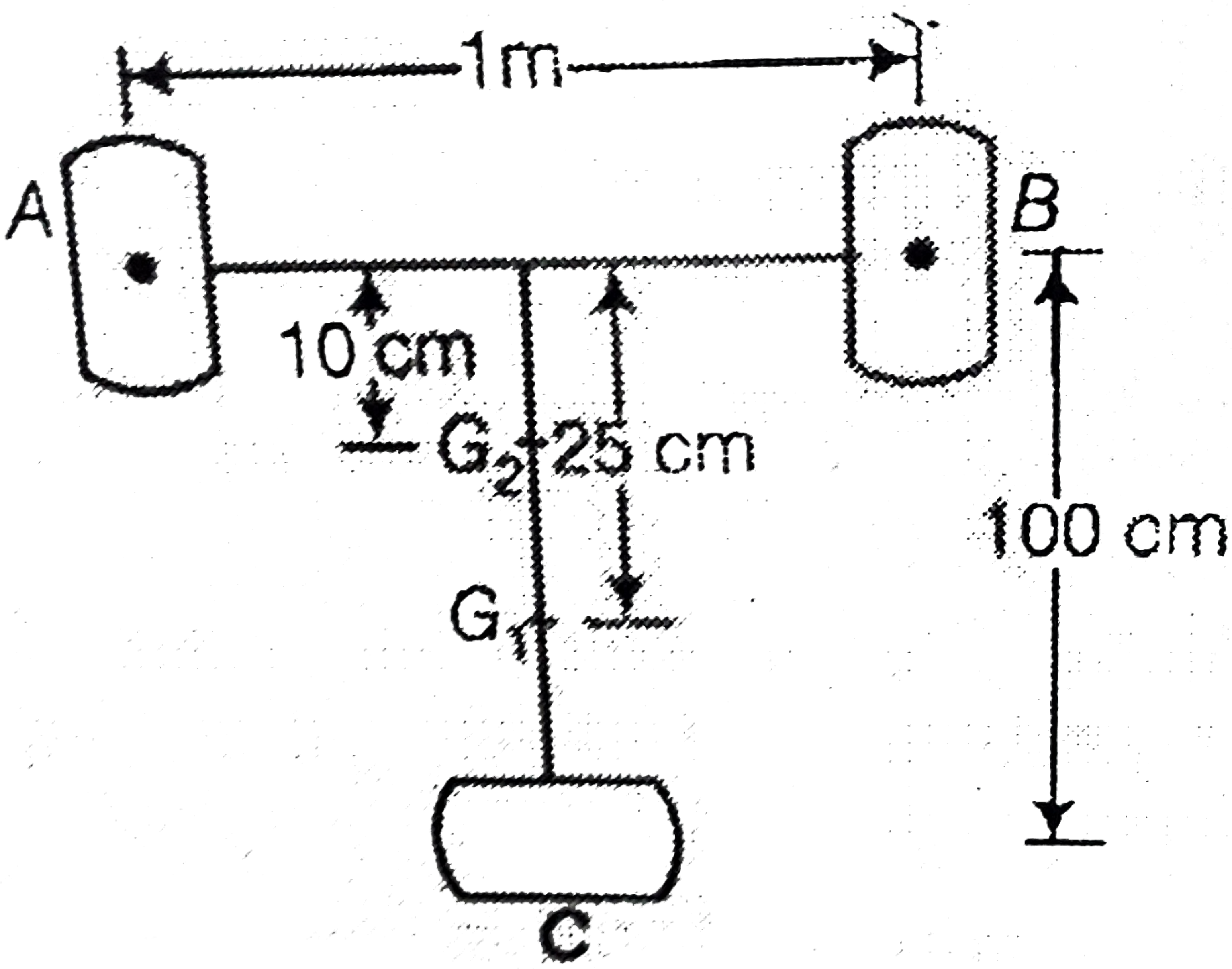
(A) 255 N

(B) 90 N

(C) 200 N

(D) 400 N

SOLUTION:



a)

In the given figure, the tricycle with two front wheels A and B with one rear wheel C with center of gravity of machine G_1 and that of rider G_2.

Now, force at $G_1 = 20 \times 10 = 200N$

Force at G_2=400 N

Let F_C = Force on ground under rear wheel C

Force at D = $F_D = 2F_A$ acts on mid-point of AB.

Taking torque about D, for equilibrium,

$F_c \times 100 = 200 \times 25$
$+ 400 \times 10 = 9000$

$\therefore$ Thrust on each front wheel is N.

$\therefore$ 2N + F_(c)
$= Total weight$
$= 600 N$
therefore$N = \frac{600 - F_C}{2}$= 255 N

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Q-59 - 11765267

A wheel of radius $6 cm$ is mounted so as to rotate about a horizontal axis through its centre. A string of negligibe mass wrapped round its circumference carries a mass of $0.2 kg$ attracted to its free end. When let fall, the mass descends through $1 m$ in $5 s$. Calculate the angular acceleration of the wheel, its moment of inertia and tension in the cord.

SOLUTION:

Here,

$r = 6cm = 6$
$\times 10^{-2} m$

$m = 0.2kg, s = 1m, t$
$= 5s$

.

If a is acceleration of falling mass, Fig.

from $s = ut + \frac{1}{2}at^2$

$1 = 0 + \frac{1}{2} \times a \times 5^2, a$
$= \frac{2}{25} m/s^2$

Angular acc.

$\alpha = \frac{a}{r} = \frac{2/25}{6 \times 10^{-2}}$
$= 1.33 rad/s^2$

Velocity after $5s$, $v = u + at$

$$= 0 + \frac{2}{25} \times 5$$

$$= 0.4m/s$$

Angular velocity

$$\omega = \frac{v}{r} = \frac{0.4}{6 \times 10^{-2}}$$

$$= \frac{20}{3} rad/s$$

According to the law of conservation of energy

Loss of $P.E =$ Increase in (K.E. of translation

$+K.E.$ of rotation)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$0.2 \times 9.8 \times 1 = \frac{1}{2}$$

$$\times 0.2(0.4)^2$$

$$+ \frac{1}{2}I\left(\frac{20}{3}\right)^2$$

$$0.196 = 0.1 \times 0.16$$
$$+ I \times \frac{400}{18}$$

$$0.180 = I \times \frac{400}{18}$$

or

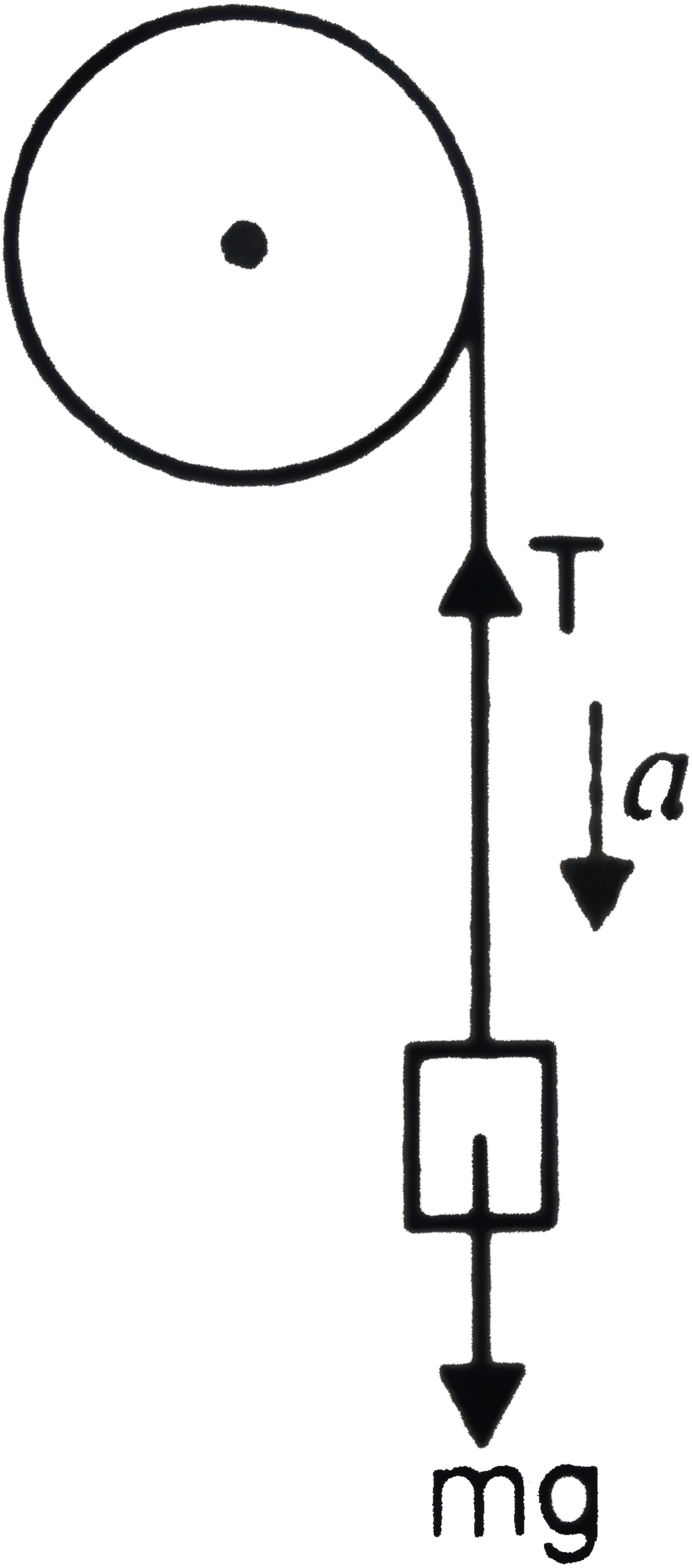
$$I = \frac{0.180 \times 18}{400}$$
$$= 8.75 \times 10^{-2} lgm^2$$

If I is tension in the string, then from $ma = mg - T$

$$T = mg - ma$$
$$= m(g - a)$$

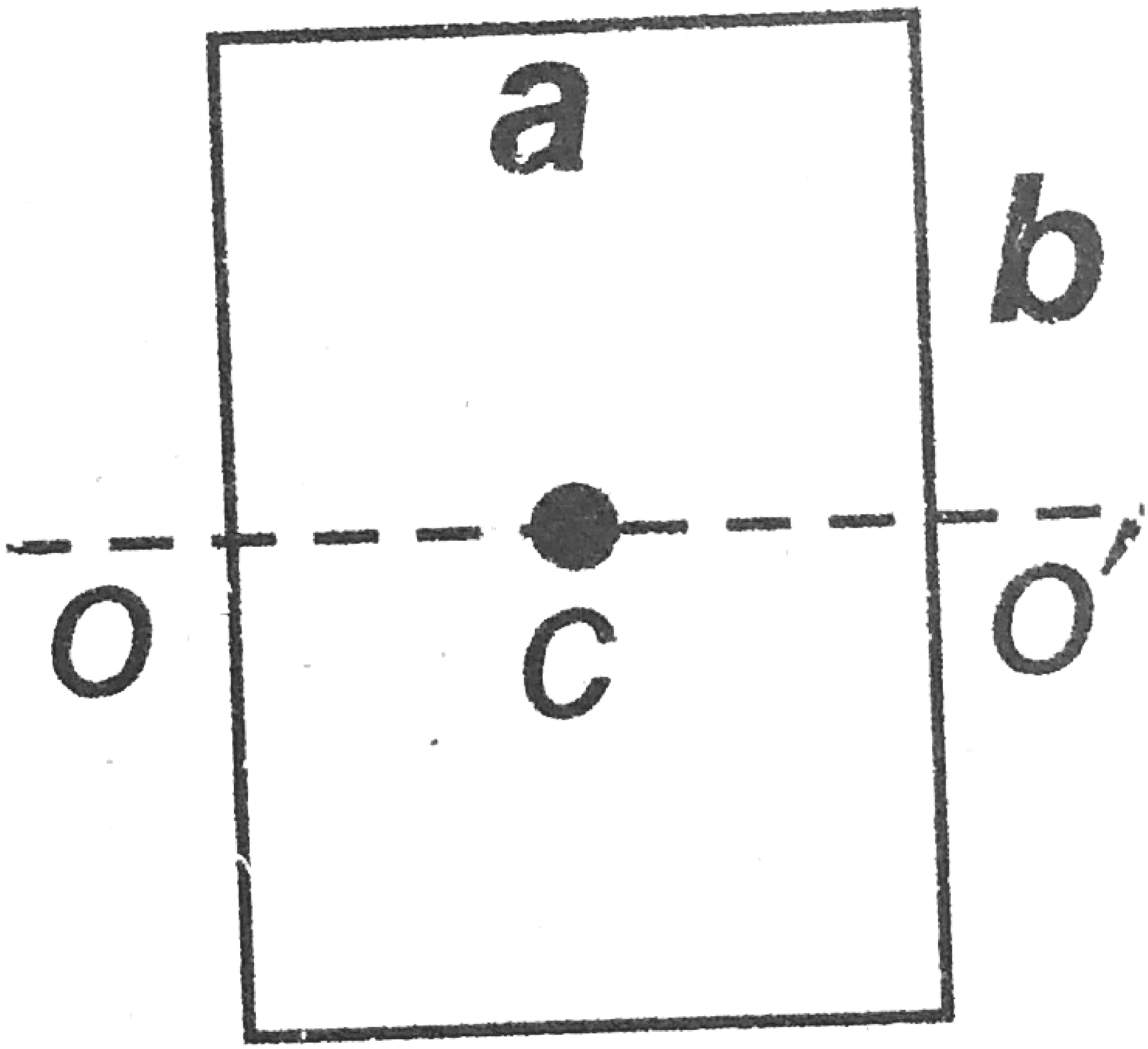
$$= 0.2(9.8 - 0.08)$$
$$= 0.2 \times 9.72$$

$$T = 1.944N$$


T
a
mg

Q-60 - 11748053

A rectangular lop has mass M and sides a and b. An axis OO' passes through the centre C of the loop and is parallel to side a (lie in the plane of the loop). Then the radius of gyration of the loop, for the axis OO' is.



(A) $\dfrac{b}{2}\sqrt{\dfrac{b+3a}{3(b+a)}}$

(B) $\dfrac{\sqrt{(a^2+b^2)}}{12}$

(C) $\sqrt{\frac{b^2+3a^2}{12}}$

(D) none of these

CORRECT ANSWER: A

SOLUTION:

(a) Let λ is linear mass density.

Mass of side a : $m_1=\lambda a$, Mass of side b : $m_2=\lambda b$

$$I_{OO'}=2\left[m_1\left(\frac{b}{2}\right)^2 + \frac{m_2b^2}{12}\right]$$

Now,

$$K_{OO'} = \sqrt{\frac{I_{oo'}}{2(m_1+m_2)}}$$

.
