Technothlon 2019 Solutions

Category: Juniors

Section A

Question 1:

Alladin thinks as the size of the chess board is 6*6 and he has 3*3 pieces, exactly two pieces should have upper right corner coloured black and two pieces with upper right corner white.
He calculates the number of cells he should recolour for each piece to make it one with upper corner black (say the number of cells needed to recolour is s) and similarly for one with upper right corner white(3*3-s).

1- 4 5
2- 7 2
3- 5 4
4- 5 4

He calculates the total number of cells needed to recolour for each combination and finds the minimum.

Ans: C) 15

Question 2:

Abu is a clever monkey. He colours squares alternately black and white in both matrices. He figures out the operation that he has applied wouldn't change the difference of sum of numbers in Black(B) and sum of numbers in white(W) boxes. Thus, matrix B is transformed from matrix A if and only if difference W-B is same.

He calculates this difference and equates it and finds that x must be 5 to satisfy the condition.

Ans: A) 5

Question 3:

Alladin observes that no matter how many times he applies the operation the numbers on a diagonal (i+j = c, c={2,3,4,5,6,7,8,9}) always stay on the same diagonal and just get jumbled. To remove the curse Aladdin must find the number of ways in which a diagonal can be jumbles, i.e. the number of permutations of the diagonal.

For c =2 3 4 5 6 7 8 9
1! 2! 3! (4!/2!) 4! 3! 2! 1!
(as 4 appears twice on this diagonal)

He gets the total number of matrices on multiplying all the above terms.

Ans: A) 41472

Question 4:

There would be four cases

1. m = 1 and n fields. It is obvious that here the answer is n.
2. m = 2 and n >= 2 fields. Here the correct formula is 2*[2*(n/4) + min(k, 2)].where k is the remainder when n is divided by 4. To see this draw the board for arbitrary n and draw all possible knight moves on it. In general, you'll see four not overlapping chains. Since you cannot place soldiers in the neighboring cells of any chain, then for a chain of length L the answer doesn't exceed (L - L/2). On the other hand, it is clear that the answer (L - L/2) is always possible since soldiers on different chains never hurt each other.
3. m >= 3 x n >= 3 fields, except the cases 3 x 3, 3 x 5, 3 x 6 and 4 x 4. Here one may use general formula m*n - (m*n)/2. Why so? It is known (or becomes known with google) that for all such fields knight tours exists. Any knight tour is just a chain of lenght m*n, so by the logic above one cannot place more than m*n - (m*n)/2 soldiers on it. On the other hand, if one makes chessboard colouring of the field, it is clear that the answer above is always achievable if one chooses cells of one colour as places for soldiers. So, formula above is proved.
4. Cases 3 x 3, 3 x 5, 3 x 6 and 4 x 4. Here we can't use the logic above to prove that the above formula is also right here. The easiest way is to verify it using brute-force or pen and paper. This concludes the solution.

So, for the given cases 2nd case will be applicable

1002x2->2*(2*250+2)= 1004
451x2->2*(2*112+2)=452
780x2->2*(2*195+0)=780
121x2->2*(2*30+1)=122

Ans: B) 2358

Question 5:

Using the above cases
53x81-> 2147
2x103-> 104
1x104-> 104

C) 2355

Section B

Question 6:

Let the number on the leftmost card be a.
Number on the next card will be a-11, a-11+8, a-11+8-7.. so on.

Sum of numbers on all these cards should be equal to sum of first 20 natural numbers which is equal to 210.

20*a+19*(-11)+18*8+17*(-7).....=210

a comes out to be equal to 13 which gives us the whole permutation

13 2 10 3 1 12 8 20 5 16 19 6 15 14 11 17 7 4 9 18
Ans: 13

Question 7:

Using the obtained permutation
Ans: 98

Question 8:

A balanced string of length n must contain m distinct letters such that m is among the divisors of n less than 27.

To minimize the number of changes made he must prefer to keep the characters that occur more frequently.

Alladin first counts the number of occurrences all 26 letters in the given string and arranges the count in descending order.
Then he finds the divisors of n (m) which are less than 27 and for each m he calculates the min number of changes to be made to obtain a balanced string having m distinct characters. For that he considers the first m numbers of the sequence and calculates sum of counts of the letters that needs to be increased, i.e. sum of (n/m)-count if count is less than n/m.
He then finds the m and corresponding count for which minimum number of changes are required.

String 1:ABAAACDHIAACKIOA (16 letters) m=2 changes=5
String 2:BBCABCDFAHABNAJHDYWGEJA (23 letters) m=1 changes=11

String 3:ABSDJAHSABDJAMDHANDHASAKIABCHD (30 letters) m=5 changes=8

Ans: 24

Question 9:

If Jasmine shoots down ak she gets ak points and has to compromise with points associated to all the balloons numbered ak+1 & ak-1, but then she can shoot all other balloons numbered ak and obtain a total of points ak*c (c= number of balloons numbered ak).
Jasmine carefully analyses the sequence of balloons and calculates points associated with each number(ak*c) and notes them down sequentially.

Ak= 1 2 3 4 5 6 7 8 9
c = 4 10 5 3 4 2 0 2 2
ak*c= 4 20 15 12 20 12 0 16 18

She maintains to variables a and b for ak. In a she stores the maximum score she can obtain by shooting balloons numbered less than or equal to ak-1 and in b she stores the maximum score she can obtain by shooting balloons numbered less than or equal to ak. Then she goes on to ak+1 and updates a to be equal to b and b to be maximum of (a+ak+1*c, b) simultaneously. She starts with 1 and a=0, b=0.

Ak= 1 2 3 4 5 7 8 9
a= 0 4 20 20 32 40 44 44 60
b=4 20 20 32 40 44 44 60 62

Answer will be the final value of b, i.e. 62

Ans: 62

Question 10:

To get the minimum total weightage Jafar must follow the following steps,
He should go through the given spell letter by letter and keep updating four variables a,b,c,d (weightage so that the statement read does not contain subsequence 'p','pi','pik', 'pika').
Initially all should be zero.
For each letter (with corresponding weightage w),
if the letter is 'p' it should be removed
a=a+w
if it is 'i' the sequence before it should not contain 'p' or the 'i' should be removed whichever is more feasible, therefore
b=min(a,b+k)
similarly for 'k' and 'a' the sequence before it should not contain 'pi' or 'pik' respectively or those letters need to be removed, whichever is more feasible
c=min(b,c+k)
d=min(c,d+k)

the final value of d after reading the whole spell is the answer

Ans: 13

Section C

Question 11:

Since the man arranges (uses) both the cubes to depict a particular date (1-31) 0,1,2 are

common numbers for both the die.

Dice 1 = 0,1,2,_,_,_

Dice 2 = 0,1,2,_,_,_

Now we need all the unit nos viz 3,4,5,6,7,8,9 and we have only 6 places left So Jasmine uses the trick that 6 can be depicted by turning the face of the die containing 9 by 180° and vice versa Since the squad wants the product-sun to be maximum.

they will choose digit 9 instead of 6.

So Dice I - 0,1,2,3,5,9

Dice 2 - 0,1,2,4,7,8

Now the squad wants thinking (hit and trial) finds that the maximum product sum comes out by keeping

0x0) + (1x1) + (2x2)+(3x4)+(5x7) + (9x8) = 124

So they place the numbers accordingly on the same colored faces. and take away the products for free from the antique shop in Paris.

Question 12: Not possible as there is a miss-print here integers should be positive.

Question 13: Points. to note – 1. The hat can be seen by other people, but not by the wearer himself, there is no exchange of words
2. There is at least one magical hat

3. Only wearers of the hat, on being sure, dunk themselves in water to remove the hat.

let's take the case of 1 magical hat. All the 49 people see 1 hat and 1 person sees no hat. Since he knows there is at least one magical hat, this sailor is sure that he is one wearing it. so, on the very first day he goes to the river dunks himself and gets the hat we removed.

Now considering the case of 2 magical hats. 48 sailors can see 2 hats and 2 persons see 1 hat. Initially (1st day) no one (out of those 2) is sure that if they wear the hat and thus, wait for one day. The next day, they see the same case, had there been only 1 magical hat, then the person who sees only 1 hat on day 1 would see no hat on day 2 (go through the above case for clarity). So, the sailors, (who see that 1 hat) become sure that they have 1 hat on their head)

Thus, on day 2 both of the sailors get their hats removed. The same logic can be extended for the given case of 7 hats. on the 7th day, all the 7 people will become sure of they bring the possessor of a magical hat and thus dunk " " in the river. "

So (1x0) + (2x0) +(3x0) +(4x0) + (5x0) +(6x0) + (7x7)=49 (ith day = no of sailors dunk in river)

The answer is 49.

Question 14:

Points to note:

1.There are more than half good people.
2. Good people say truth, bad people lie.

Suppose we chose a person and ask him about , the adjacent(or any other) person whether the person is good/bad.
If the person being asked says ‘NO’ then
-If he is good then other person is bad.
-If he is bad, then he lies , so the other person is good.

If the answer is no and we remove the two persons ,the one who we asked and the one who was asked about (if he is good or not), We remove 1 good and 1 bad person. So the statement that there are more than half good people still remains valid.
Having figured this out , Aladdin tells this to Genie , also starts calculating the worst case ,i.e. minimum no. of steps to find at least 1 good person.

NOTE: It should take care that after these minimum steps you should be sure that You have 1 good person in your reach , or you can point one good person with surety.
Genie on thinking analysis the situation when the answer is Yes, suppose person 1 says 2 is good then he asks 2 about 3. If the answer is 'NO', he removes 2 and 3 else he continually asking 3 and so on. This way he makes a chain in which every person thinks that his adjacent one is good. So, either they all lie or they all tell the truth. Since there are more than half good people. Ones the chain is more than half (>=45) we can be sure that All of them are good.
Genie is not sure enough and thinks of one move way, if he continues removing persons (1 good 1 bad) in pairs then if he is left with only two people or one, He can be sure that they are good. As the persons removed contain equal no. of good and bad people and to make no. of good more than half the remaining 2 or 1 have to be good.
He tells this Aladdin and on discussion, they figure out that each person (except 1 and the last) is being asked about their nature.
For ex. We ask P1 about P2- YES
P2 about P3- NO-remove P2 &P3
P1 about P4- YES (we are not bothered for now)
So we see Pn-1 would be asked about Pn would not be asked about(So second logic Genie thinks of)
At max n-2 People would be asked about their nature , after which we will be sure of at least one being good.
Aladdin finds the answers as 87 with the help of Genei.

Ans: 87

Section D

Question 15:
Sum=39 points, so we have a sum of either 13 or 3(39=13*3 only). For 3 it is impossible to find 3 positive integers(placed on back of Aladdin, Genie and Jasmine) that sum to 3, so the sum is 13 and they played 3 games.
Ans: A) 3

Question 16:

We prove that if there is an even number of candies, 2n, then the first player can win. This is true for 2 candies. Assume it is true for 2(n-1) candies. We will prove it is true for 2n candies. Assume that the first player can't win. Then, if he eats half of the candies, there will be n candies left. If he was losing, the position should now be a winning position for the second player. Consider the possibilities when he takes one candy. The other person is left with 2n-1 candies. The other person can either take a.)1 candy. Then the first player is left with a position with 2n-2 candies, which is winning by inductive hypothesis.

b.) half the candy. Then the first player is left with a position with n candies, which is winning.

Therefore, the first player wins. By retrograde analysis we can in fact decide who wins if starting with any N>1 candy. If we write N=2^m i + 1, with i odd and $m>= 0, then the first player wins if m is even, and the second player wins when m is odd. This is because first player wins when N is even, as shown above, and then because of the simple observation that 2^{m+1} i + 1 is winning for a player if and only if 2^{m} i + 1 is losing for that player (and of course, also the other way around).

Aesc)64

Question 17:

a) can. 72 > 27 > *17 > 459 > 954 > 91 > 19 >*2017 > 38323
b) cannot as 11 divides 341 but not 9241
c) cannot as 11 does not divide 1002 but it divides 11022
d) cannot 11 divides 9614 but not 7205427

Ans: A)72 to 38323

Question 18: [BONUS QUESTION]

Suppose there are r red points(houses) among some n points(houses).
Since any pair of them can lie on at most two blue-centered unit circles, it means that the number b of blue points(houses) can be at most 2 * rC2 = r(r - 1). Since b + r = n, this leads to condition r + r (r - 1) = r^2 >= n, i.e. r >= ceil (sqrt{n}) , so b <= n - ceil (sqrt{n})

A simple model is given by r = ceil (sqrt{n}) red points of coordinates Ri(ri, 0), with 0 < ri < 2, for all i = 1,2...r.
Take n - r blue points among those r(r - 1) given by all pairs (i, j), 1 <= i < j <= ceil (sqrt{n}), and of coordinates B{i,j}(x{i,j}, b{i,j}) and B'{i,j}(x{i,j}, -b{i,j}), with
Xi,j = (ri + rj) /2 and b{i,j} = sqrt{1 - ({ri - rj}/2) ^2 }
It is trivial to check that on a unit circle of center B{i,j} or B'{i,j} lie points Ri and Rj, and only them.

The choices made warrant that n - r <= r(r - 1), since it comes to n <= (least_integer(sqrt{n}))^2, and that ({ri - rj}/2) ^2 < 1.
For the given n = 2009, the answer is thus that the largest possible number of blue points is 2009 - least_integer(sqrt{2009})= 2009 - 45 = 1964(houses)

Ans: D) 20

Section E

Question 19:

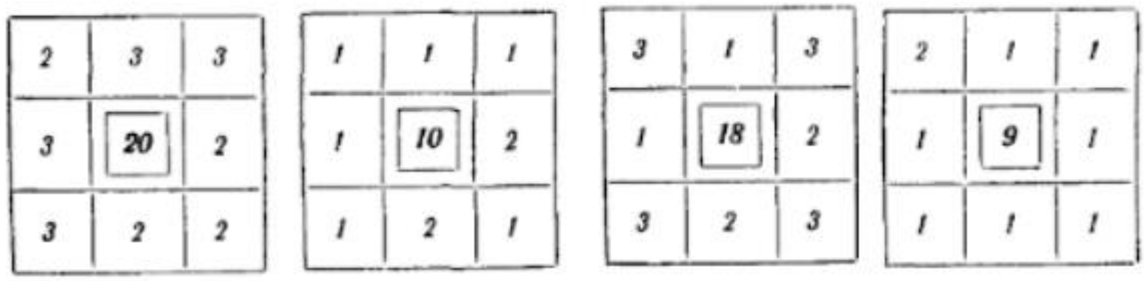
Clearly n=1 works. We claim that it is the largest positive integer for which a coloring is possible.

Suppose n>1. Note that any n+1 consecutive sides can be seen. Consider two consecutive line segments x,y with colours A,B.here colour A,B,C represent soldiers of the 3 kingdoms. Any n+1 consecutive sides containing these line segments must only have the colours A,B. The union of all such sets is the collection of all the sides of the regular 2n+1-gon, excluding the side directly opposite the vertices shared by x,y. Hence this side must be coloured C, and is the only side coloured C. Take the n+1 consecutive sides with C as its starting edge and x as its ending edge; all the sides in between must be A. Similarly, take the n+1 consecutive sides with C as its starting edge and y as its ending edge; all sides in between must be coloured B. Now take a set of n+1 consecutive sides with C not at one of the ends; for n>1 it contains both sides coloured A,B, invalidating the colouring.

Ans: B) 1

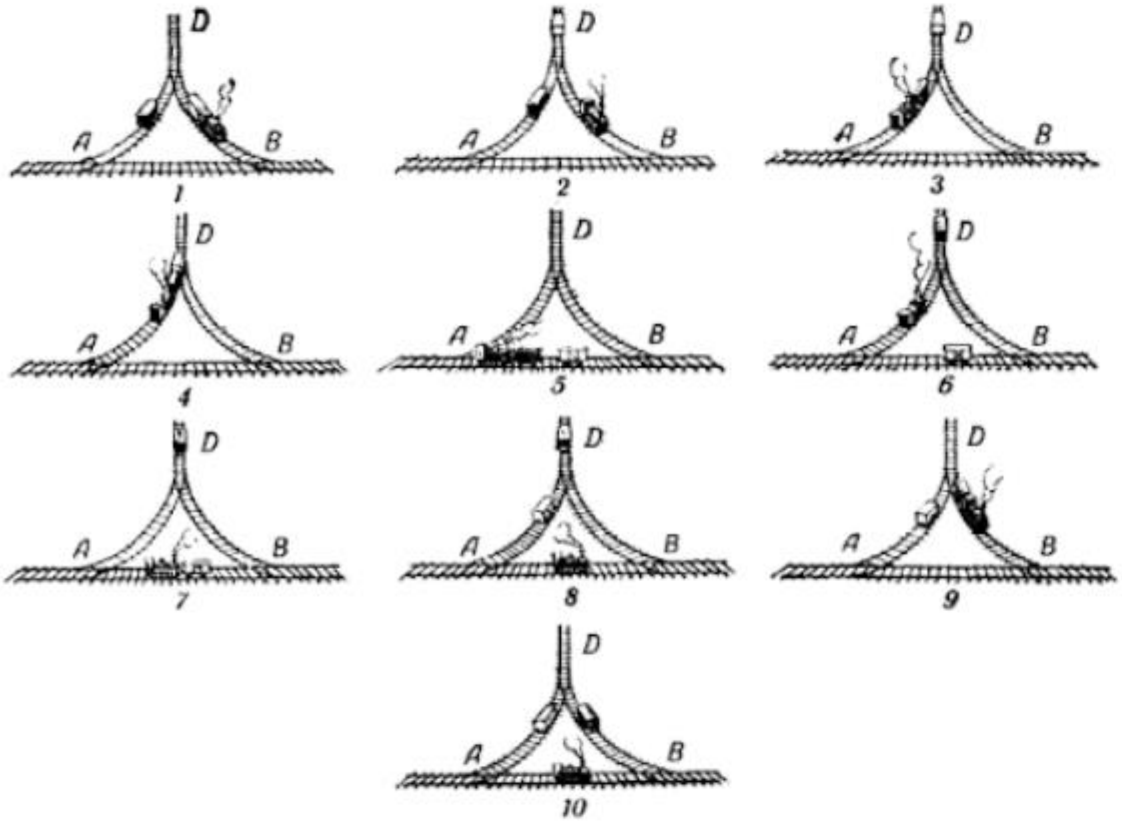
Question 20:

It follows from condition 3 that from 22 through 44 soldiers can be housed (for minimum place all the soldiers in the corner sections and for maximum place all the soldiers in middle sections). But the number of soldiers must be a multiple of 3 (condition 4). Thus the number can be 24, 27, 30, 33, 36, 39, or 42. Further, trial shows 24 soldiers cannot be housed 11 to a side (condition 3) without leaving empty sections (condition 1), and that 33, 36, 39, or 42 soldiers cannot be housed 11 to a side without placing more than 3 soldiers in some sections (condition 2). By elimination, 30 soldiers were expected, and 27 arrived. The diagrams show how they were housed. (In both pairs of diagrams, the second floor is on the left, and the first floor on the right.)



Ans: B) 30

Question 21:

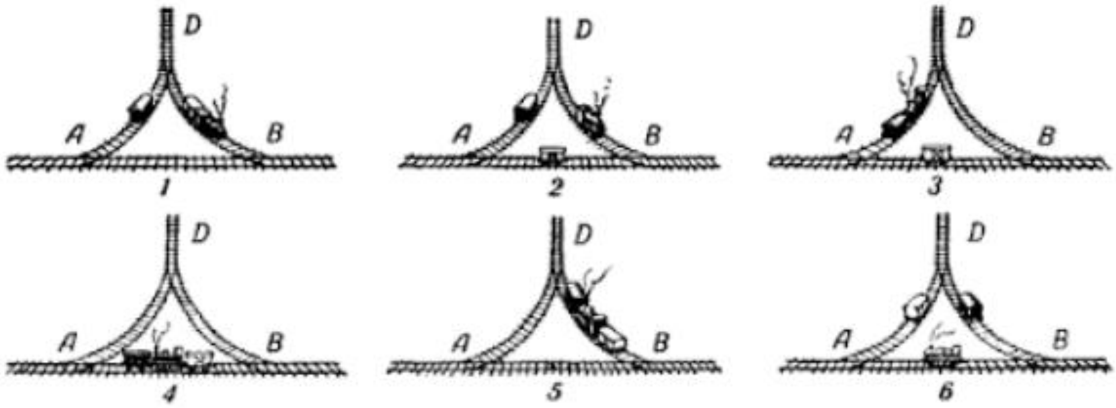


1. Alladin backs into BD and couples on the white car.
2. He backs the white car into D and uncouples, then moves out along DB.
3. He passes B, backs up past/4, enters AD, and couples on the black car.
4. He pushes the black car forward, couples it on the white car, and starts backing out of AD, pulling both cars.
5. He backs past/4, goes partway forward to B, and uncouples the white car.
6. Leaving the white car on AB, he goes backward past A pulling the black car, then pushes it up AD into D and uncouples. He starts backing out of AD.
7. He backs past/4 and moves forward on AB till he can couple on the white car.
8. He backs past A, pushes the white car onto AD and uncouples, then backs past A and moves forward partway to B.
9. He passes B, backs into BD and couples on the black car, pulling it forward along DB.
10. Uncoupling the black car on BD, he passes B and backs up till he is again halfway between A and B, facing right.

(There are at least two other solutions in 10 moves.)

Ans: B) 16

Question 22:



1. Alladin backs into BD and couples on the white car.
2. He pulls the white car past B, backs up along BA, and uncouples the white car. Leaving it on AB, he moves forward past B, then backs into BD.
3. Moving in and out of D, he moves forward along DA and couples on the black car.
4. He pushes the black car past A, then backs up along AB and couples on the white car.
5. Sandwiched between the cars, he backs pasts and enters BD. He uncouples the black car.
6. Leaving the black car on BD, he backs out pushing the white car pasts, pulls it along AB Past A, pushes it back into AD where he uncouples the white car, moves out of AD past A, then backs up until he is again halfway between A and B, but facing left.

Ans: D) 6

Category: Hauts

Section A

Question 1: Answer = 9

Let's take a general case of two such sequences. (C_i and T_i)

Consider the difference array $d_1, d_2, \ldots, d_{n-1}$, where $d_i = c_{i+1} - c_i$. Let's see what happens if we apply synchronization.

Pick an arbitrary index j and transform c_j to $c'_j = c_{j+1} + c_{j-1} - c_j$. Then:

- $d'_{j-1} = c'_j - c_{j-1} = (c_{j+1} + c_{j-1} - c_j) - c_j - 1 = c_{j+1} - c_j = d_j$
- $d'_j = c_{j+1} - c'_j = c_{j+1} - (c_{j+1} + c_{j-1} - c_j) = c_j - c_j - 1 = d_{j-1}$

In other words, synchronization simply exchanges two adjacent differences. That means that it's sufficient to check whether the difference arrays of two sets of stones are equal.
For Black Widow, we get : 1 , 2, -5, 6, -3, -2
For Hawkeye, we get : 6, 2, ***x-12 , 10-x,*** -5, -2, 3.
Thus, Case1) **10 - x = 1 and x - 12 = -3** are the two equations obtained from the comparison which gives value of x as **9**. The other case yields x = 13 (Not in options)

Question 2: Answer: Yes, No, No, No.

Net cutting heads in each case:

1. 22 - 15 = 7
2. 67 - 25 = 42
3. 1 - 15 = -14
4. 62 - 34 = 28

On observing the number of heads being cut and growing on the dragon(assuming both occur at once) we see that the difference is always a multiple of 7. In each case, consider 4 cases of cutting heads (22,67,1,62 as the final blow) and then we check if we can reduce the dragon heads to that number;

Initial Heads	Cut 22 in final	Cut 67 in final	Cut 1 in final	Cut 62 in final
88	66	***21***	87	26
135	113	68	134	73
192	170	125	191	130
152	130	85	151	190

We observe in above table that only 21 is a multiple of 7 and thus the dragon can be killed. (Cutting 22 each time and 15 grow back = total cutting 7) **88 => 81 => 74 => 67** and then cut 67 to kill the dragon.

In all other cases, the dragon can not be killed.

Q3) and Q4)

Considering the height of wall as sequences in each case;
If any two consecutive numbers are having the same parity(parity means even or odd) then remove both the numbers from the sequence. If at the end the sequence has 0 or 1 element then it is possible to do so and in other case it cant be done.

If it is possible then either minimum bricks required is (n*max-sum)/2 is it is integer else (n*(max+1)-sum)/2.
Where max is the maximum number of the sequence.
Sum is the total sum of the sequence.

Question 3: Answer: 1456
The numbers are: O, O, O, E, E, O, E, O, O
Total sum of existing wall sizes: 87+451+91+34+234+127+56+9+67 = 1156
Case 1) If the final height of wall is to be 451(Max number in given seq)
{Here dividing by 2 because each brick size is 2}
More bricks required = (451*9 - 1156)/2 = 1451.5 (Not integer).
Case 2) If final height is 452
More bricks required = (452*9 - 1156)/2 = 1456.

Question 4: Answer: infinity
The numbers are: E, E, O, O, O, E, E, E. Thus, can't be done

Question 5: Answer: Shield
First, money spent by each company in that order is 500*0.03 = 15 (This needs to be reduced to find the profits in each case)

From Clue 4 its clear that the Hammer Industries sold at 0.15 and Stark sold at 0.08
From Clue 2, Shield has sold its products at 0.1 and Oscorp at 0.75. Using trial and error, allotting the number of products sold (Keeping Clue 3 in mind), we can find that:
(0.1 * Shield) - 15 = (0.075 * Oscorp) - 15 + 3.8
Shield = 413 with profit 26.30 , Oscorp = 500 with profit 22.50.
(It can be observed that the Shield has more products sale and also the more profit)
From Clue 1, We get that least amount of profit is 26.30 - 11.30 = 15.00, which is satisfied by Stark by selling 375 goods(Observing existing cases).
Mr. Shield sold 413 at $0.1 each for a $26.30 profit.
Ms. Oscorp sold 500 at $0.075, for a profit of $22.50.
Ms. Hammer sold 219 at $0.15 each, for a profit of $17.85.
Mr. Stark sold 375 at $0.08 for a profit of $ 15.00.

Section B

Question 8: Ans 2

Solution. Suppose that we have an arrangement satisfying the problem conditions. Divide the board into 2×2 pieces; we call these pieces *blocks*. Each block can contain not more than one king (otherwise these two kings would attack each other); hence, by the pigeonhole principle each block must contain exactly one king.

Now assign to each block a letter T or B if a king is placed in its top or bottom half, respectively. Similarly, assign to each block a letter L or R if a king stands in its left or right half. So we define *T-blocks*, *B-blocks*, *L-blocks*, and *R-blocks*. We also combine the letters; we call a block *a TL-block* if it is simultaneously T-block and L-block. Similarly we define *TR-blocks*, *BL-blocks*, and *BR-blocks*. The arrangement of blocks determines uniquely the arrangement of kings; so in the rest of the solution we consider the 50×50 system of blocks (see Fig. 1). We identify the blocks by their coordinate pairs; the pair (i, j), where $1 \leq i, j \leq 50$, refers to the jth block in the ith row (or the ith block in the jth column). The upper-left block is $(1, 1)$.

The system of blocks has the following properties..

(i′) If (i, j) is a B-block then $(i + 1, j)$ is a B-block: otherwise the kings in these two blocks can take each other. Similarly: if (i, j) is a T-block then $(i - 1, j)$ is a T-block; if (i, j) is an L-block then $(i, j - 1)$ is an L-block; if (i, j) is an R-block then $(i, j + 1)$ is an R-block.

(ii′) Each column contains exactly 25 L-blocks and 25 R-blocks, and each row contains exactly 25 T-blocks and 25 B-blocks. In particular, the total number of L-blocks (or R-blocks, or T-blocks, or B-blocks) is equal to $25 \cdot 50 = 1250$.

Consider any B-block of the form $(1, j)$. By (i′), all blocks in the jth column are B-blocks; so we call such a column *B-column*. By (ii′), we have 25 B-blocks in the first row, so we obtain 25 B-columns. These 25 B-columns contain 1250 B-blocks, hence all blocks in the remaining columns are T-blocks, and we obtain 25 *T-columns*. Similarly, there are exactly 25 *L-rows* and exactly 25 *R-rows*.

Now consider an arbitrary pair of a T-column and a neighboring B-column (columns with numbers j and $j + 1$).

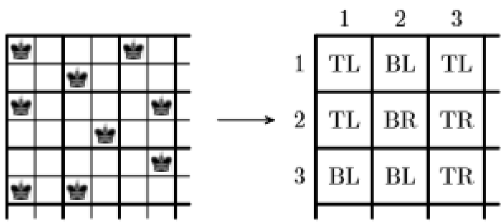


Fig. 1

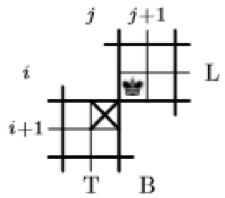


Fig. 2

Case 1. Suppose that the jth column is a T-column, and the $(j + 1)$th column is a B-column. Consider some index i such that the ith row is an L-row; then $(i, j + 1)$ is a BL-block. Therefore, $(i + 1, j)$ cannot be a TR-block (see Fig. 2), hence $(i + 1, j)$ is a TL-block, thus the

$(i+1)$th row is an L-row. Now, choosing the ith row to be the topmost L-row, we successively obtain that all rows from the ith to the 50th are L-rows. Since we have exactly 25 L-rows, it follows that the rows from the 1st to the 25th are R-rows, and the rows from the 26th to the 50th are L-rows.

Now consider the neighboring R-row and L-row (that are the rows with numbers 25 and 26). Replacing in the previous reasoning rows by columns and vice versa, the columns from the 1st to the 25th are T-columns, and the columns from the 26th to the 50th are B-columns. So we have a unique arrangement of blocks that leads to the arrangement of kings satisfying the condition of the problem (see Fig. 3).

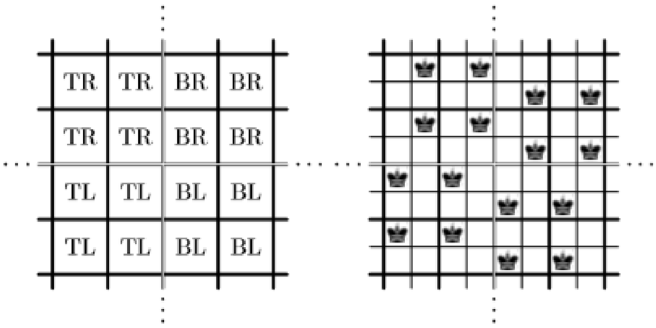


Fig. 3

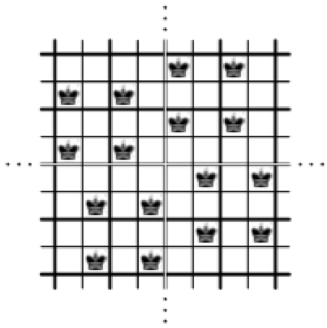


Fig. 4

Case 2. Suppose that the jth column is a B-column, and the $(j+1)$th column is a T-column. Repeating the arguments from Case 1, we obtain that the rows from the 1st to the 25th are L-rows (and all other rows are R-rows), the columns from the 1st to the 25th are B-columns (and all other columns are T-columns), so we find exactly one more arrangement of kings (see Fig. 4).

Question 9:

The instruction of black widow would be reversed exactly by the opposite instruction of Hawkeye so the delivery boy will comes backs to his initial location so black widow will never win.
Ans) 0

Section C

Q10) and Q11):

Consider the number of spikes in Strange's turn, noting down the winning positions and losing positions:(Win positions are those which can lead to Lose positions of Wong and Losing positions are those which can led to the Win position for Wong in all cases i.e. removing 1,3,or 4 spikes)

1. Win (Remove 1 spike)
2. Lose
3. Win (Remove 1 or 3 spikes)
4. Win (Remove 4 spikes)
5. Win (Remove 3 spikes)
6. Win (Remove 4 spikes)
7. Lose and the same sequence repeats from starting.

Question 10: Answer: Strange, 1 or 3 spikes

Extending the above sequence till 31, we observe for n=31, it is Win position for Strange by removing 1 or 3 spikes.

Question 11: Answer: 49
For less than 50 spikes, 49 is the Lose position for Strange.

Question 12: TO BE DELETED

Q13) and Q14)
Let us first consider the first question, as it is significantly easier to answer than the second. Knowing that Rocket is the most accurate and therefore most deadly dueler, we can assume that Rocket would be the first target for Gamora and Drax. Rocket knows that Gamora is more accurate and deadly than Drax, so Rocket would first take aim at Gamora. Furthermore, Drax will not be targeted until either Rocket or Gamora is dead, so it is in Drax's best interest to not interfere with the duel between Rocket and Gamora. In summary, Rocket would aim at Gamora and Gamora at Rocket; Drax would refrain from firing until Rocket or Gamora is killed. Drax is not initially targeted and therefore has the greatest chance of survival.

Question 13: Answer: Drax.
Since Drax has the highest chance of surviving, Thanos must have disguised as Drax.

Question 14: Answer: 47/90

Let's start with Rocket: Rocket has a ½ chance of getting off the first shot in his duel with Gamora (based on who draws a better straw), in which case he will kill Gamora and survive that round. If Gamora shoots before Rocket (a ½ chance), he has a 1/5 chance of missing, so Rocket has a (1/2) * (1/5) chance of surviving. Rocket has a (1/2) + (1/10) = 3/5 chance of surviving the first round of shots. Now Drax will shoot at Rocket, with a ½ chance. If Drax misses, Rocket will shoot him and win. We multiply Rocket's chances for the first round (3/5) with his chances for the second round (1/2) to obtain 3/10, or 30%.
Gamora's case is more complicated because if he wins the first round, he has the potential for an indefinitely long exchange of shots with Drax. Because Rocket's survival chances against Gamora are 3/5, Gamora's chances against Rocket are 2/5 (1 – 3/5). The math gets hairy between Gamora and Drax – I am going to simplify by providing Gardner's insight. The duel between Gamora and Drax becomes a repeating decimal of the series from n = 1 to infinity of 4/(10^n) (for those less mathematically inclined, it looks like 4/10 + 4/100 + 4/1000 . . .). This is equivalent to 4/9. Multiplying Gamora's first and second round chances, we obtain (2/5) * (4/9) = 8/45.
To solve for Drax, we could use the same process or simply subtract the other two probabilities from 1 and obtain 47/90.

Rocket = ⅓; Gamora = 8/45; Drax = 47/90

The entire truel can be conveniently graphed by using the tree diagram shown.

Section D

Question 15: Answer : y in first round: Warmachine, z in second round: Vision.

Total sum = 39 points, so we have a sum of either 13 or 3. for 3 it is impossible to find 3 positive integers that sum to 3, so the sum is 13 and they played 3 games.
Consider, hitting x as win, z as lose and y as draw.
The minimum number is at least 1, the maximum number is at most 10. (let's call highest number win, lowest loss, middle draw)

Assume Iron man lost once, then Warmachine can have lost at most twice, and we cannot find such numbers. So, Iron man has 2 wins and a draw. Vision and C both have a loss and we still have to split {win,loss,draw,draw} over Vision, Warmachine. Since Vision = Warmachine +1 and Vision won once we have:
Iron man = wwd = > wwl
Vision = llw = > llw
Warmachine = ldd = > ddd

Vision won the last game, so lost the others and Warmachine drawed all the others, so Warmachine also drawed the first one.

Question 16: Ans: 0 and 1
Note that the set of sequences of moves the fire makes is in bijective correspondence with the set of strings of Xs and Ys of length 15, where X denotes a move which is either counter clockwise or inward along a spoke and Y denotes a move which is either clockwise or outward along a spoke. (The proof of this basically boils down to the fact that which one depends on whether the fire is on the inner wheel or the outer wheel.) Now the condition that the fire ends at A implies that the difference between the number of Xs and the number of Ys is a multiple of 5, and so we must have either 4, 9 or 14 Xs within the first fourteen moves with the last move being an X . This implies the answer is

14C4+14C9+14C14=3004.
So, the remainders with 2, 3, 7 are : 0, 1, 1 respectively

Q17) and Q18)

The main claim is that if a player is forced to reduce the minimum number of outriders over all piles, then they lose. Intuitively, everytime a player reduces the minimum, the other player has a move that does not reduce the minimum, and if a player isn't forced to reduce the minimum, they have a move that will force the other player to reduce the minimum.
Let m be the minimum number of outriders in a squad and let x be the number of squads with m outriders. Rocket can win if and only if x<=n/2 . Let's call the positions where Rocket can win as winning positions and all other positions as losing positions. To show why this works, we need to show from a winning position, we can reach some losing positions and from a losing position, we can reach only a winning position.
If we are at a winning position, there are at least n/2 squads that have strictly more than m outriders, so we can choose any arbitrary subset of them and reduce them to m outriders. This is now a losing position. If we are at a losing position, no matter what we do, we must include a squad of size m in our chosen subset. If m is zero, this means we have no available moves. Otherwise, the minimum will strictly decrease, but only at pst n/2 squads (from the squads that we chose) can reach that new minimum. Thus losing positions can reach only winning positions. If the frequency of the minimum number is <= n/2, then Rocket wins, otherwise Thor wins.

Question 17: Answer: Rocket

The frequency of minimum number “6” is 3, which is less than 5. Thus, Rocket wins.

Question 18: Answer: min: 32 and max: 39

If Thor has to land last attack and win, then the frequency of minimum number must be greater than 5. Thus, at least three of a,b,c,d must be 8 and the other number must be >=8 and <=15.
Minimum: 8 + 8 + 8 + 8 = 32
Maximum: 8 + 8 + 8 + 15 = 39

Section

Q19) and Q20)

It follows from condition 3 that from 22 through 44 soldiers can be housed. But the number of soldiers must be a multiple of 3 (condition 4). Thus the number can be 24, 27, 30, 33, 36, 39, or 42. Further, trial shows 24 soldiers cannot be housed 11 to a side (condition 3) without leaving empty sections (condition 1), and that 33, 36, 39, or 42 soldiers cannot be housed 11 to a side without placing more than 3 soldiers in some sections (condition 2). By elimination, 30 soldiers were expected, and 27 arrived. The diagrams show how they were housed. (In both pairs of diagrams, the second floor is on the left, and the first floor on the right.)

Question 19: Answer: 30
Question 20: Answer: 31
M = 2 + 3+ 2+3+1+1+1+1 = 14
N = 3+3+3+3+2+1+1+1 = 17

Question 21: Answer: 21

Yes: (1,2),(3, 5), (4, 7), (6, 10), (8, 13), (9, 15), (11, 18), (12, 20), . . .
This series of number pairs is closely related to Fibonacci numbers and the golden ratio. The first pair differ by 1, the second pair by 2, and the nth by n. Every positive integer appears once and only once in the series of pairs. These pairs are called WYTHOFF PAIRS.

Since, max number of stones is 15;
The losing positions for Black Widow are (1,2),(3, 5), (4, 7), (6, 10), (8, 13), (9, 15).

Sum of differences: 1 + 2 + 3 + 4 + 5 + 6 = 21

Question 22: Answer: 4

Number the top coin in the pyramid 1, the coins in the next row 2 and 3, and those in the bottom row as 4, 5 and 6. The following four moves are one of the solutions: Move 1 to touch 2 and 4, move 4 to touch 5 and 6, move 5 to touch 1 and 2 below, move 1 to touch 4 and 5.